

DEGENERATE MATRICES METHODS BY SPLINES FOR BOUNDARY VALUES PROBLEMS OF ORDINARY DIFFERENTIAL EQUATIONS

MIHAILS BELOVS¹ and TEODORS CIRULIS²

Institute of Mathematics of Latvian Academy of Sciences and University of Latvia

Akadēmijas laukums 1, LV-1524, Rīga, Latvia

E-mail: {¹mihails.belovs, ²teodors.cirulis}@lu.lv

Degenerate matrices (DM) methods for initial values problems of ODE based on the use of pseudoinverse matrices for derivatives lead to very effective algorithms which depend on two independent parameters: the step h and the order $N + 2$ of pseudoinverse matrix. An accuracy of the solution can be increased with both the diminution of h and the enlargement of N [1]. The direct use of simplest matrices for derivatives in order to solve boundary values problems of ODE leads to algorithms dependent on only one parameter $N + 2$ – the order of the matrix for derivatives. Such algorithms are more difficult to apply than ones for the initial values problem of ODE. An analysis of numerical solutions for initial value problems of ODE shows that these solutions present splines of the order $N + 1$ and defect N which are constructed simultaneously with the solution. Therefore, we suggest to use such splines depending on the concrete boundary values problem also for solving of boundary values problems of ODE. Usually the order of splines is chosen as $10 \div 100$. Simple matrices for derivatives with nodes as zeroes of classical orthogonal polynomials are used essentially to construct the splines.

Main results. A new algorithm for solving boundary-value problems of ordinary differential equations is introduced with following. Main features:

- Additional limitations on values of function and its derivatives in the boundary points are obtained from the condition of compatibility of the numerical approximating system.
- Together with the boundary conditions (they can be also non-linear) these limitations give a system of equations. (In the case of the second order ODE it has 4 equations and 4 unknowns). We obtain the values of the unknown function and its derivatives in the boundary points from this system.
- Finally a solution of the boundary-value problem can be done in various ways.
- The schema of the decomposition for the boundary-value problem's solution is built.
- The method can be translated also to cases of singular boundary-value problems, boundary-value problems for the systems of ODE, spectral problems and boundary-value problems for non-linear equations.

REFERENCES

- [1] T. Cirulis and O. Lietuviētis. Degenerate matrix method for solving nonlinear systems of differential equations. *Math. Model. Anal.*, **3**, 1998, 43 – 56 .