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NONSTATIONARY STAGE OF QUASI-CHERENKOV BEAM INSTABILITY IN PERIODICAL STRUCTURES

K. BATRAKOV[1] and S. SYTOVA[2]

Institute for Nuclear Problems, Belarusian State University

11 Bobruiskaya, 220050, Minsk, Belarus

E-mail: {[1]batrakov, [2]sytova}@inp.minsk.by

This contribution is devoted to further investigation by methods of mathematical modeling of quasi-Cherenkov beam instability in periodical structures. Such physical mechanism serves as a basis for Volume Free Electron Lasers (VFEL) that are progressing rapidly. Let us consider the following system of equations describing nonlinear stage of quasi-Cherenkov instability:

$$\frac{\partial E}{\partial t} + a_1\frac{\partial E}{\partial z} + b_{11}E + b_{12}E_\tau = \Phi\int_0^{2\pi}\frac{2\pi - p}{8\pi^2}\left(\exp(-i\Theta(t,z,p) + \exp(-i\Theta(t,z,-p)\right)dp,$$

$$\frac{\partial E_\tau}{\partial t} + a_2\frac{\partial E_\tau}{\partial z} + b_{21}E + b_{22}E_\tau = 0,$$

$$\frac{d^2\Theta(t,z,p)}{dz^2} = \Psi\left(k - \frac{d\Theta(t,z,p)}{dz}\right)^3 \operatorname{Re}\left(E(t - z/u, z)\exp(i\Theta(t,z,p))\right),$$

$$E|_{z=0} = E_0, \quad E_\tau|_{z=L} = E_1, \quad E|_{t=0} = 0, \quad E_\tau|_{t=0} = 0, \quad \Theta(t,0,p) = p, \quad \frac{d\Theta(t,0,p)}{dz} = k - \omega/u,$$

where $t > 0$, $z \in [0, L]$, $p \in [-2\pi, 2\pi]$. This system is a system of integro-differential equations. In addition to temporal argument we have two independent arguments: spatial coordinate z and initial electron phase p. Amplitudes of electromagnetic fields $E(t,z)$, $E_\tau(t,z)$ and coefficients a, b and Φ are complex-valued. Function $\Theta(t,z,p)$ describes phase of electron beam relative to electromagnetic wave. Θ and coefficient Ψ are real. k is a projection of wave vector on axis z. ω is a field frequency. u is an initial electron beam velocity.

Introducing uniform grids on t, z and p

$$\omega_t = \{t_l = lh_t,\ l = 0,1,\ldots\}, \quad \omega_z = \{z_m = mh_z,\ m = 0,1,\ldots,M,\ Mh_z = L\},$$

$$\omega_p = \{p_j = h_p j,\ j = -N,\ldots,-1,0,1,\ldots,N,\ h_p N = 2\pi\},$$

we can write the following difference scheme:

$$\widehat{\Theta}^j_{\bar{z}z} = \Psi\left(k - \widehat{\Theta}^j_{\overset{\circ}{z}}\right)^3 \operatorname{Re}\left(\widetilde{E}\exp(i\Theta^j)\right), \text{ where } \widetilde{E} = E(t_l - \alpha t_m, z_m),\ \alpha = h_z/(h_t u),$$

$$E_t + a_1\widehat{E}_{\bar{z}} + b_{11}\widehat{E} + b_{12}\widehat{E}_\tau = \Phi\sum_{j=0}^{N} c_j\left(\exp(-i\widehat{\Theta}^j) + \exp(-i\widehat{\Theta}^{-j})\right), \quad E_{\tau t} + a_2\widehat{E}_{\tau z} + b_{21}\widehat{E} + b_{22}\widehat{E}_\tau = 0.$$

c_j are coefficients of quadrature formula. We take entier of α. When $t_l - \alpha t_m < 0$, $\widetilde{E} = E(0, z_m)$.

Results of numerical experiments carried out are discussed. In Bragg geometry of VFEL the regime of bifurcations was investigated in detail.