Mathematical Modelling and Analysis
Abstracts of the 9th International Conference MMA2004, May 29-31, 2004, Jūrmala, Latvia


# ON POSITIVE CO-MONOTONE HISTOPOLATION BY COMBINED QUARTIC SPLINES

SVETLANA ASMUSS

*Institute of Mathematics of Latvian Academy of Sciences and University of Latvia*

Akademijas laukums 1, LV-1524, Riga, Latvia

E-mail: asmuss@lanet.lv

Let $F = \{f_i\}_{i=1}^n$ be a histogram describing a finite sample with the frequency $f_i$ for the class interval $[t_{i-1}, t_i]$, $i = 1, 2, \dots, n$. We consider the histopolation problem of construction of a spline $s$, which satisfies the area-matching conditions

$$\int_{t_{i-1}}^{t_i} s(t)\,dt = f_i h_i, \quad h_i = t_i - t_{i-1}, \quad i = 1, 2, \dots, n, \tag{1}$$

and is positive and co-monotone according to the given histogram $F$:

$$s(t) \geq 0, \quad s'(t) f_i' \geq 0 \quad \text{for} \quad t \in [t_{i-1}, t_i], \quad i = 1, 2, \dots, n, \tag{2}$$

where $f_i' = 1$ $(f_i' = -1)$ if the histogram is found to be increasing (decreasing) in $[t_{i-1}, t_i]$. Usual quartic histosplines [1], which belong to the space $S(T, A^1)$ defined by the operators

$$T : W_2^2[a,b] \to W_2^0[a,b],\ Tx = x'' \quad \text{and} \quad A^1 : W_2^2[a,b] \to R^n,\ A_i^1 x = \int_{t_{i-1}}^{t_i} x(t)\,dt\ :$$

$$S(T, A^1) = \{s \in W_2^2[a,b]\ :< Ts, Tx >= 0 \text{ for all } x \in Ker A^1\ \}, \quad [a,b] = [t_0, t_n],$$

and satisfy (1), do not preserve positivity or monotonicity of the given data (2).

Our approach uses combined splines [2] of the space $S(T, A^1 \times A^2)$ formed by quartic histosplines of $S(T, A^1)$ and qubic Hermite splines of the space $S(T, A^2)$ given by the same operator $T$ and the operator

$$A^2 : W_2^2[a,b] \to R^{2n+2}, \quad A_i^2 x = x(t_i), \quad A_{n+1+i}^2 x = x'(t_i) \quad i = 0, 1, \dots, n.$$

We investigate splines of this combined space, use them for solving of the histopolation problem (1) with free parameters of the Hermite interpolation and present a method to choose these parameters so that the solution preserves the shape of the data in the sense (2).

**REFERENCES**

[1] I.J. Schoenberg. Splines and histograms. *Spline functions and approximation theory. Proc. Sympos. Univ. Alberta. Internat. Ser. Numer. Math*, **21** (1973) 277-327.

[2] E.A. Rimša. Combined splines. *Continuous functions on topological spaces*, Riga (1986) 155-158.