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I-TOPOLOGICAL SPACES

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An ideal on a set X is a non-empty collection of subsets closed under taking subsets and finite unions. Two subsets are said to be equivalent if their symmetric difference is an element of the ideal. We are interested in an i-topological space.

Definition 1. An i-topological space is a set X together with a collection $\mathcal{T}$ of subsets and an ideal $\mathcal{I}$ on X that satisfy the following conditions:

(T1) $\varnothing$ and X are in $\mathcal{T}$;
(T2) the union of an arbitrary number of sets in $\mathcal{T}$ is equivalent to some set in $\mathcal{T}$;
(T3) the intersection of a finite number of sets in $\mathcal{T}$ is equivalent to some set in $\mathcal{T}$;
(T4) $\mathcal{T} \cap \mathcal{I} = \{\varnothing\}$.

Given an i-topological space $(X, \mathcal{T}, \mathcal{I})$. A subset $E \subseteq X$ is said to be *with empty $\mathcal{I}$-interior* if $U \setminus E \notin \mathcal{I}$ for any non-empty $U \in \mathcal{T}$. A subset $A \subseteq X$ is said to be *nowhere $\mathcal{I}$-dense* if for each subset $E \subseteq X$ with empty $\mathcal{I}$-interior the union $A \cup E$ is with empty $\mathcal{I}$-interior as well. We use the notation $c(\mathcal{I})$ for a set of all nowhere $\mathcal{I}$-dense sets.

Theorem 2. *Given an i-topological space* $(X, \mathcal{T}, \mathcal{I})$.

(1) $c(\mathcal{I})$ *is an ideal on* X *and* $c(\mathcal{I}) \cap \mathcal{T} = \varnothing$;
(2) $\mathcal{I} \subseteq c(\mathcal{I})$;
(3) $c(c(\mathcal{I})) = c(\mathcal{I})$.

Given an i-topological space $(X, \mathcal{T}, \mathcal{I})$. A subset $A \subseteq X$ is said to be *nd-set* if for each $a \in A$ there exists $U \in \mathcal{T}$ such that $a \in U$ and $U \cap A \in \mathcal{I}$. An i-topological space is said to be *trivial* if the following condition is satisfied for each $V \in \mathcal{T}$ and for each $\mathcal{U} \subseteq \mathcal{T}$: if $V \cap U \in \mathcal{I}$ for every $U \in \mathcal{U}$ then $V \cap (\bigcup_{U \in \mathcal{U}} U) \in \mathcal{I}$ as well.

Theorem 3. *An i-topological space* $(X, \mathcal{T}, \mathcal{I})$ *is trivial if and only if each nd-set is nowhere $\mathcal{I}$-dense.*

The idea of nd-set is found in the compatible ideal's definition in [1; 2].

Currently we are working on the existence of a homeomorphic (up to elements of an ideal) topological space for each trivial i-topological space. We will discuss our progress on this problem.

REFERENCES

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[2] D. Janković and T.R. Hamlett. Compatible extensions of ideals. *Boll. Un. Mat. Ital. B(7)*, **6** (3), 1992, 453 – 465.