FREE PLANAR GRAPHS ON TORUS

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Graph on plane, projective plane or torus is free planar if it remains embeddable in corresponding surface after arbitrary edge is added. Free planar graph is called *ladder-augmentable* if two edges in it may be replaced with arbitrary large ladder-graph [3] and graph will remain to be free planar with respect to surface where it is embedded. There are only two nontrivial 3-connected free planar graphs on sphere, i.e. wheel graph and envelope graph, where only the first can be unboundedly augmented with a *punkah* like augment [3] retaining its 3-connectivity and freeness on the sphere. Envelope graph may be augmented with ladder like augment only when it is on the Mobius strip. Similarly on torus, if 3-connected toroidal free planar graph is ladder-augmentable then it should be projective planar, except in some special cases [3], e. g. shown in Fig.1. The graph in Fig.1 is one of minimal forbidden graphs for projective plane (graph D_{17} in [2] and the only one which becomes planar when edge is eliminated from it). Perhaps it is the only one which is projective non-planar and free ladder-augmentable on torus.

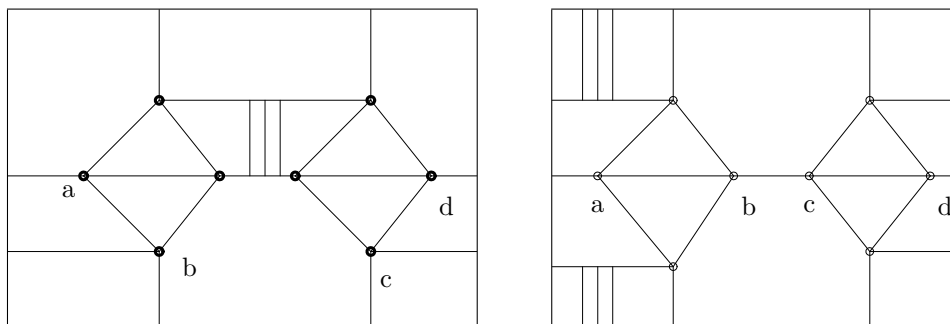$K_{3,3}$ and K_5 are ladder-augmentable because they both are projective planar [3].

Figure 1. Counterexample: forbidden graph for projective plane that is free ladder-augmentable on torus, in too embeddings. Free planarity is easy discernable.

Examples of other nontrivial free toroidal graphs are shown in [3].

REFERENCES

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[2] H.H. Glover, J.P. Huneke and Wang Chin San. 103 graphs that are irreducible for the projective plane. *J. Combin. Theory Ser. B*, **27** , 1979, 332 – 370.

[3] D. Zeps. Free planar graphs on torus: examining 3-connected graphs for unbounded augmentability. *To be published.*