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MULTIPLE SOLUTIONS OF STURM-LIOUVILLE TYPE BOUNDARY VALUE PROBLEMS

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The problem

$$x'' = f(t,\, x,\, x') \tag{1}$$

$$x(0)\cos\alpha - x'(0)\sin\alpha = 0, \qquad x(1)\cos\beta - x'(1)\sin\beta = 0, \tag{2}$$

is considered, where $f \in C(I \times R^2,\, R)$, $I = [0,\, 1]$.

We use the following approach. Suppose that equation (1) can be written in the equivalent quasi-linear form

$$\big(L_2 x\big)(t) = F_1(t,\, x,\, x'), \tag{3}$$

where $\big(L_2 x\big)(t)$ is a nonresonant linear part and F_1 is continuous and bounded. Then the problem (3), (2) has a solution $x_1(t)$, which solves also the problem (1), (2). If (1) can be reduced to another quasi-linear equation

$$\big(l_2 x\big)(t) = F_2(t,\, x,\, x'), \tag{4}$$

then the problem (1), (2) has another solution $x_2(t)$. In this way one can obtain the multiplicity results.

As an example, consider the problem

$$x'' = -2mx' - \lambda^2\, |x|^p \operatorname{sign} x, \tag{5}$$

$$x(0) - x'(0) = 0, \qquad x(1) + x'(1) = 0, \tag{6}$$

where $m,\, \lambda \in \mathbb{R}$, $\quad p > 0$, $\quad p \neq 1$. Equation (5) can be reduced to

$$\frac{d}{dt}\big(e^{2mt}\, x'\big) + e^{2mt}\, k^2 x = e^{2mt}\, F_k(x), \tag{7}$$

where $F_k(x) := f_k\big(\delta(-N_k,\ x,\ N_k)\big)$, $\ f_k(x) := k^2 x - \lambda^2\, |x|^p \operatorname{sign} x$, $\ N_k > 0$. Then if the inequality $\Gamma_k \cdot M_k < N_k$ holds, where $M_k = \max\limits_{x \in \mathbb{R}} |F_k(x)|$, $\Gamma_k = \max\limits_{0 \le t, s \le 1} \big|G(t,s)\big|$ (and $G(t,s)$ is the respective Green's function), then the original problem has a solution $x_k(t)$. The multiplicity result follows.

Proposition 1. *Given $m \in \mathbb{R}$ and a positive integer r one can find $\varepsilon > 0$ such that if p satisfies $|p-1| < \varepsilon$, then there exist at least $r+1$ different solutions of the problem* (5), (6).

REFERENCES

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