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ON A CATEGORY OF L-TOPOLOGICAL SPACES ON GLOBAL L-VALUED SETS

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Let $L = (L, \leq, \wedge, \vee, *)$ be a GL-monoid [1]. A global L-valued equality on a set X is a mapping $E : X \times X \to L$ such that:

1) $E(x,x) = 1 \quad \forall x, y \in X$;
2) $E(x,y) = E(y,x) \quad \forall x, y \in X$;
3) $E(x,y) * E(y,z) \leq E(x,z) \quad \forall x, y, z \in X$.

An L-valued set is a pair (X, E) where X is a set and E is an global L-valued equality on it.

An L-subset A of an L-valued set (X, E) is called *extensional* if

$$\bigvee_{x \in X} A(x) * E(x, x') \leq \ A(x') \ \ \forall x' \in X.$$

Let $\mathcal{L}(X)$ denote the family of all extensional L-subsets of X.

By an L-fuzzy topology on a global L-valued set (X, E) we call a mapping $T : \mathcal{L}(X) \to L$ s. t.

1) $\mathcal{T}(1_X) = \mathcal{T}(0_X) = 1$;
2) $\mathcal{T}(U \wedge V) \geq \mathcal{T}(U) \wedge \mathcal{T}(V) \quad \forall U, V \in \mathcal{L}(X)$;
3) $\mathcal{T}(\bigvee_{i \in \mathcal{I}}(U_i) \geq \bigwedge_{i \in \mathcal{I}} \mathcal{T}(U_i) \quad \forall \{U_i \mid i \in \mathcal{I}\} \subset \mathcal{L}(X)$.

The triple $(X, E, \mathcal{T})$ is called an L-fuzzy L-valued topological space.

Note that in case E is crisp the above definition reduces to the definition of an L-fuzzy topological space in the sense of [2].

Let L-$FTOP(L)$ denote the category whose objects are L-fuzzy L-valued topological spaces and whose morphisms are extensional continuous mappings between them. (The continuity of $f : (X, E_X, \mathcal{T}_X) \to (Y, E_Y, \mathcal{T}_Y)$ means that $\mathcal{T}_X(f^{\leftarrow}(V)) \geq \mathcal{T}_Y(V)$ for each $V \in (\mathcal{L})(X)$.)

Our aim here is to discuss some properties of the category L-$FTOP(L)$ and its objects. In particular, it will be shown that L-$FTOP(L)$ is topological over the category $SET(L)$ of global L-valued sets. Besides some relations between this category and some other categories will be studied.

REFERENCES

[1] U. Höhle. $M-$valued sets and sheaves over integral commutative cl-monoids. In: *Applications of category theory to fuzzy subsets (Linz, 1989)*, S.E. Rodabaugh, E.P. Klement and U. Höhle (Eds.), Kluwer Acad., Dordrecht, 1992, 33 – 72.

[2] A. Šostak. Two decades of fuzzy topology: basic ideas, notions and results. *Russian Math. Surveys*, **44** (6), 1989, 125 – 186.