Acta Societatis Mathematicae Latviensis
Abstracts of the 5th Latvian Mathematical Conference, April 6–7, 2004, Daugavpils, Latvia
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MEASURE OF CONVERGENCE AS AN L-TOPOLOGICAL PROPERTY

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There is an increasing number of papers where classical mathematical notions, such as convergence, continuity, differentiability, etc., are allowed to be valid (or not) not absolutely, but to a certain extent. It is the aim of this talk to sketch how such concepts can be interpreted and studied in the appropriate context of L-topologies. Here we confine ourselves to the property of convergence of a sequence in a metric space (X, ρ).

Recall first that the (upper) measure of convergence of a sequence $(a_n)_{n\in\mathbb{N}}$ to a point x can be defined by $\mu(x) = \sup_{n\in\mathbb{N}} \left(\inf_{k\geq n} \frac{\rho(x,a_k)}{1+\rho(x,a_k)}\right)$, see e.g. [1]

Further, it is known that by setting $E_\rho(x,y) := \frac{1}{1+\rho(x,y)}$ an L-valued equality E_ρ on X is defined, where $L := [0,1]$ viewed as an MV-algebra, i.e. with the Łukasiewicz t-norm $*$. Now we can rewrite $\mu(x) = \sup_{n\in\mathbb{N}} \left(\inf_{k\geq n} E_\rho(x,a_k)\right)$.

To get a topological interpretation of this number, note that the set $\mathcal{T} := \mathcal{T}(X,E)$ of all E_ρ-*extensional* mappings $U : X \to L$ determines an L-topology on a set X. On its turn, a sequence (a_n) gives rise to an L-filter $\mathcal{F} : L^X \to L$ in the sense of [2] as follows:

$$\mathcal{F}(A) = \sup_{n\in\mathbb{N}} \left(\inf_{k\geq n} A(a_k)\right), \quad A \in L^X.$$

The promised interpretation is given now by the equality

$$\mu(x) = (\lim \mathcal{F})(x),$$

where $\lim \mathcal{F}$ is the limit of the L-filter with respect to L-topology $\mathcal{T}$ [3] that is

$$\lim \mathcal{F}(x) = \sup \{\alpha \in L \mid \forall v \in \mathcal{T} : v(x) * \alpha \leq \mathcal{F}(v)\}.$$

Moreover, applying local L-valued equalities (instead of the global one E_ρ) we can have even a deeper insight into this problem.

Acknowledgements The author is indebted to U. Höhle for drawing his attention to these problems.

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