

## UNIT ROOT TEST FOR BOUNDED PROCESSES

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Despite the extensive literature on modelling non-stationary economic time series, a rarely discussed topic is how to interpret and analyze time series whose behavior can be well approximated by means of integrated process (I(1)) but are "limited" in the sense that their range is constrained by fixed bounds [2]. Range constraints represent the standard framework in the context of target zone exchange rates and, more generally, of time series which are subjected to control.

In our study we consider an integrated AR(1) process

$$y_t = \rho y_{t-1} + \varepsilon_t \quad (1)$$

with reflection from boundaries  $-a \leq y_t \leq a$ , where  $\rho = 1$  and  $\varepsilon_t$  is a Gaussian white-noise process ( $\varepsilon_t \equiv i.i.d.N(0, \sigma^2)$ ). It is known that when range constraints are not applied, the OLS estimate of  $\rho$  is biased to be below its true value of unity [1]. For this reason Dickey and Fuller [1] proposed to test for a presence of unit root using the following regression:

$$\Delta y_t = \gamma y_{t-1} + \varepsilon_t, \quad (2)$$

where  $\gamma = \rho - 1$ . Equation (2) can be estimated directly using OLS. Testing of  $\gamma = 0$  for regression equation (2) is equivalent to testing of  $\rho = 1$  for equation (1). Test statistics has a form  $\tau = \frac{\hat{\gamma}}{s(\hat{\gamma})}$ , where  $s(\hat{\gamma})$  is standard error of  $\hat{\gamma}$ . In case when there are no range constraints, Dickey-Fuller test can be performed using Dickey-Fuller table of critical values, but for case of constraints no critical values for this test were published.

In our study we used Monte Carlo simulations to derive distribution of statistics  $\tau$  in standard Dickey-Fuller test (2) for a random walk process (1) (with  $\rho = 1$ ) subjected to range constraints for different values of bounds  $a$ . Our results show that distribution of statistics  $\tau$  for limited I(1) process approaches classical Dickey-Fuller distribution (see [1]) as the bound  $a$  tends to infinity. The obtained critical values for the statistics  $\tau$  allow to test bounded processes for presence of unit root.

### REFERENCES

- [1] D.A. Dickey and W.A. Fuller. Distribution of estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association*, (74), 1979, 427 – 431.
- [2] G. Cavaliere. The asymptotic distribution of the Dickey-Fuller statistics under a non-negativity constraint. *Econometric Theory*, (19), 2003, P/S series, P.03.3.2.