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TWO-POINT NONLINEAR BOUNDARY VALUE PROBLEMS: QUASILINEARIZATION AND TYPES OF SOLUTIONS

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We consider the two-point nonlinear boundary value problems (BVP) with respect to the solvability and types of solutions. If the scalar BVP is given in the form

$$x'' = f(t, x, x'), \tag{1}$$

$$x(0) = 0, \quad x(1) = 0, \tag{2}$$

then it has a solution if the right side f is continuous and bounded (Picard theorem). If equation is given in the form

$$(l_2 x)(t) = x'' + p(t)x' + q(t)x = f(t, x, x'), \tag{3}$$

where p, q, f are continuous and f is bounded, then the problem (3), (2) is solvable if the linear part $(l_2 x)(t)$ is nonresonant.

Theorem 1. *Suppose that: 1) a solution y of the Cauchy problem $(l_2 x)(t) = 0$, $y(0) = 0$, $y'(0) = 1$ has exactly i zeros ($i = 0, 1, \ldots$) in the interval $(0, 1)$ and $y(1) \neq 0$; 2) f in (3) is bounded. Then there exists a solution ξ of the problem (3), (2) such that the difference $x(t; \alpha) - \xi(t)$ for α small enough also has exactly i zeros ($i = 0, 1, \ldots$) in the interval $(0, 1)$ and $x(1; \alpha) - \xi(1) \neq 0$, where $x(t; \alpha)$ are solutions of (3), $x(0; \alpha) = 0$, $x'(0; \alpha) = \xi'(0) + \alpha$.*

The result above means that quasi-linear problems have solutions with the same oscillatory type as the one for the linear part of the equation.

In what follows we discuss the method of upper and lower functions, its efficiency and shortcomings and finally present an alternative method of quasilinearization, which provides also the necessary and sufficient conditions for solvability and is suitable for equations of different oscillatory types.

Theorem 2. *Suppose there exists a function $u \in C^2(0, 1)$ and $N > 0$ such that $u'' = f(t, u, u') + \varepsilon(t)$ and $\Gamma \cdot M(N) \leq N$, where $\Gamma = \max\{|G(t, s)| : 0 \leq t, s \leq 1\}$, $G(t, s)$ is the Green function for the problem $x'' = 0$, (2),*

$$M(N) = \max\{|f(t, x, x') - f(t, u(t), u'(t)) - \varepsilon(t)| : 0 \leq t \leq 1,\ |x - u(t)| \leq N,\ |x' - u'(t)| \leq N\}.$$

These conditions are necessary and sufficient for the solvability of the problem (1), (2).