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Λ-CONVEX FUNCTIONS AND LAMINATE CLOSURE

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For given smooth coercive strictly convex functions $F_i\colon \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, N$, by the laminate (constructed from $F_1, \ldots, F_N$ in direction e and in given proportions $\alpha_1, \ldots, \alpha_N$ respectively) we understand a function $\mathcal{F}\colon \mathbb{R}^n \to \mathbb{R}$, which corresponds to the local energy density of the composite constructed from materials with energy densities F_i, $i = 1, \ldots, N$, and proportions $\alpha_1, \ldots, \alpha_N$ respectively distributed in layers orthogonal to e. Analytically the function $\mathcal{F}$ has the representation

$$\mathcal{F}(z) = \lim_{\varepsilon \to 0} \inf \left\{ \int_K \sum_{i=1}^N \chi_i \left(\frac{1}{\varepsilon} \langle x, e \rangle \right) F_i(\nabla u(x) + z)\, dx \;\middle|\; u \in W_p^1(K), u \text{ is } K-\text{periodic} \right\},$$

where K is the unit cube $(0,1)^n$ and χ_i are the characteristic functions of regions occupied by materials with energy density F_i respectively.

We show that if the set M of strictly convex smooth functions $\mathcal{F}\colon \mathbb{R}^n \to \mathbb{R}$ is closed with respect to lamination, i.e. M contains all laminates constructed by means of functions $\mathcal{F}$ from M, then the function $L\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$,

$$L(z, \xi) = \inf \left\{ \mathcal{F}(z) + \mathcal{F}^*(\xi) \;\middle|\; \mathcal{F} \in M \right\}$$

is Λ-convex with respect to the cone $\Lambda = \left\{ (z, \xi) \in \mathbb{R}^n \times \mathbb{R}^n \;\middle|\; \langle z, \xi \rangle = 0 \right\}$, i.e.

$$\begin{gathered} L(\lambda z' + (1-\lambda) z'', \lambda \xi' + (1-\lambda)\xi'') \le \lambda L(z', \xi') + (1-\lambda) L(z'', \xi'') \\ \text{whenever } (z' - z'', \xi' - \xi'') \in \Lambda \text{ and } 0 \le \lambda \le 1. \end{gathered} \tag{1}$$

Here by $\mathcal{F}^*$ we denote the conjugate to $\mathcal{F}$ function.

On the other hand, if a coercive continuous function $L\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ satisfies (1) then the set

$$\left\{ \mathcal{F}\colon \mathbb{R}^n \to \mathbb{R} \;\middle|\; \mathcal{F} \text{ is strictly convex, } \mathcal{F}(z) + \mathcal{F}^*(\xi) \ge L(z, \xi)\ \forall z, \xi \in \mathbb{R}^n \right\}$$

is convex and closed with respect to lamination.