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THE SECOND-ORDER EQUATION OF DUFFING TYPE

SVETLANA OGORODNIKOVA

University of Daugavpils

Parādes iela 1, LV-5400, Daugavpils, Latvia

E-mail: oglana@tvnet.lv

We consider the second-order equation of Duffing type ([1], p.12)

$$x'' = -\alpha x + x^3, \quad \alpha > 0, \tag{1}$$

together with the Dirichlet boundary conditions

$$x(0) = 0, \quad x(1) = 0. \tag{2}$$

Let N(α) be the number of nontrivial solutions to the problem (1), (2). Our aim is to give estimations of the number N(α). Standard analysis of the phase portrait shows that

$$N(\alpha) = 2i, \quad \text{if} \quad i^2\pi^2 < \alpha < (i+1)^2\pi^2, \quad i = 0, 1, \dots$$

For the equation

$$x'' = f(x), \tag{3}$$

where f is a continuous function such that

$$f(x_1) = f(0) = f(x_2) = 0, \quad f < 0,\ x < x_1,\ 0 < x < x_2; \quad f > 0,\ x_1 < x < 0,\ x > x_2,$$

the analogous estimates in terms of $f'(0)$ look as:

$$N = 2i, \quad if \quad i^2\pi^2 < |f'_x(0)| < (i+1)^2\pi^2, \qquad i = 0, 1, \dots$$

The results are generalized to the case of the "shifted" Duffing equation

$$x'' = -\alpha(x-a) + (x-a)^3,$$

where $x = 0$ is not a singular point yet, and to the case of the equation (3), where $f \in \mathcal{C}(\mathbb{R})$,

$$f(x_1) = f(a) = f(x_2) = 0, \quad f < 0,\ x < x_1,\ a < x < x_2; \quad f > 0,\ x_1 < x < a,\ x > x_2.$$

REFERENCES

[1] R. Seydel. *Practical bifurcation and stability analysis. From equilibrium to chaos.* Springer, New York, 1994.