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ON THE SOME PROPERTIES OF SOLUTIONS OF PSEUDOPARABOLIC EQUATIONS

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Various physical phenomena have led to a study of third order pseudoparabolic equations [1] $\Delta u + \eta \Delta u_t - u_t = 0$, where the constant η is positive. Let $G(t_0, t_1)$ be a domain in the Euclidean space $R^{n+1}_{x,t} = (x_1, \ldots, x_n, t)$, bounded by the planes $t = t_0$, $t = t_1$ and by the surface $\Gamma \in C^1$ which is not tangent to planes $t = const$ and belongs to the layer $\{x, t; x \in R^n_x, t_0 < t < t_1\}$. We assume that for all $\tau \in (t_0, t_1)$ $\Omega(\tau) = G \cap \{x, t : t = \tau\}$ is a bounded domain of R^n_x. In the domain $G(t_0, t_1)$ we consider the pseudoparabolic equation of the form

$$-\left(m^{ij}(x,t)u_{tx_j}\right)_{x_i} + m(x,t)u_t - \left(l^{ij}(x,t)u_{x_j}\right)_{x_i} + l^i(x,t)u_{x_i} + l(x,t)u = 0. \tag{1}$$

We assume for simplicity that the coefficients are sufficiently smooth functions in $G(t_0, t_1)$. Suppose that for any $(x,t) \in G(t_0, t_1)$ and any $\xi = (\xi_1, \ldots, \xi_n) \in R^n_\xi$

$m^{ij} = m^{ji},\ m^{ij}\xi_j\xi_i \geq 0,\ m \geq 0,$

$(l^{ij} - 2^{-1}m^{ij}_t)\xi_j\xi_i \geq \alpha\,|\xi|^2,\ \alpha = const. > 0,\ (l - 2^{-1}m_t - 2^{-1}l^i_{x_i}) \geq 0.$

We use the notations $V(u) = (l^{ij} - 2^{-1}m^{ij}_t)u_{x_j}u_{x_i} + (l - 2^{-1}m_t - 2^{-1}l^i_{x_i})u^2$, $R(u) = m^{ij}u_{x_j}u_{x_i} + mu^2$.

Theorem 1. *Let $u(x,t)$ be a weak solution of equation (1) in $G(t_0,t_1)$ satisfying boundary condition ($u\big|_\Gamma = 0$). Then the following estimates hold*

$$\int\limits_{G(\tau,t_1)} V(u)dxdt \leq \exp\{-\int\limits_{t_0}^{\tau} 2\lambda(s)ds\} \int\limits_{G(t_0,t_1)} V(u)dxdt,$$

$$\int\limits_{G(\tau,t_1)} V(u)dxdt \leq 2^{-1}\exp\{-\int\limits_{t_0}^{\tau} 2\lambda(s)ds\} \int\limits_{\Omega(t_0)} R(u)dx,$$

where $0 < \lambda(s) \leq \inf\limits_{v\in\Psi} (\int\limits_{\Omega(s)} V(v)dx)(\int\limits_{\Omega(s)} R(v)dx)^{-1}$ and Ψ - is the set of functions $v(x)$ such that $v(x) \in C^\infty\left(\overline{\Omega}(s)\right)$, $v \equiv 0$ on $\overline{\Omega}(s) \cap \Gamma$.

From these estimates follow uniqueness theorems for weak solutions of problems without the initial conditions in classes of growing functions.

REFERENCES

[1] R.E. Showalter and T.W. Ting. Pseudoparabolic partial differential equations. *SIAM J. Math. Anal.*, **1** (1), 1970, 1 - 26.