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ABOUT ONE REGULARIZATION METHOD FOR THE SOLUTION OF THE FIRST KIND OPERATOR EQUATIONS WITH INEXACT DATA

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In this paper we consider an operator equation of the first kind

$$Az = u, \tag{1}$$

where H is a Hilbert space, the operator $A\colon H \to H$ is linear, self-conjugate, positive and completely continuous, $u \in AH$ is a given element and $z \in H$ is the unknown element.

In this paper it is assumed that the equation (1) is solvable for all $u \in H$, i.e. $AH = H$. Let it is known a'priory information that the equation (1) has the exact solution z^p under the given exactly initial data $\{A^p; u^p\}$ where $A = A^p\colon H \to H$ is the exact operator, and $u = u^p \in H$ is the exact right-hand side of the equation (1), i.e. $A^p z^p = u^p$. In the work [1] is considered the method for construction of the approximate solution of the equation (1) which is stable against the small changes of the initial information when only right side of the equation (1) was inexact, but the operator was assumed exactly known, i.e. instead of $u = u^p$ there was $\left\{u^\delta; \delta\right\}$ such that $\|u^p - u^\delta\| \le \delta$.

The following theorems are proved in [1].

THEOREM 1. *Let z^p be an exact solution of (1) for $u = u^p \in H$. Then the iterative process*

$$\begin{cases} z_{n+1}^\delta = (E - \theta \cdot A)\, z_n^\delta + \theta \cdot u^\delta, \quad n \ge 0 \\ z_0^\delta = 0 \end{cases} \tag{2}$$

converges to the exact solution z^p of equation (1) in the norm of given Hilbert space H, if the number of iterations n is chosen from the condition $\sqrt{n} \cdot \delta \to 0$ as $n \to \infty$, $\delta \to 0$.

Moreover, under the condition

$$0 < \theta \le \frac{4}{3 \cdot \|A^p\|_H}$$

for the iterative process (1) the following error estimate holds:

$$\|z^p - z_n^\delta\|_H \le (2 \cdot n \cdot m \cdot \theta \cdot e)^{-\frac{1}{2}} \cdot \|z^p\|_H + \left(\frac{4}{3} \cdot \frac{n}{m} \cdot \theta\right)^{\frac{1}{2}} \cdot \delta, \quad n \ge 1,$$

where $m = \inf_{\|x\|=1} (A^p x, x)$.

THEOREM 2. *The optimal error estimate for iterative process* (2) *under the conditions of Theorem 1 has the form*

$$\|z^p - z_n^\delta\|_H^{optimal} \le \left(\frac{13}{10}\right)^{\frac{1}{4}} \cdot \left(2 \cdot \delta \cdot m^{-1} \cdot M \cdot \|z^p\|_H\right)^{\frac{1}{2}},$$

and this bound is reached for

$$n^{optimal} = \left[\sqrt{\frac{10}{13}} \cdot \frac{1}{\theta \cdot \delta} \cdot \|z^p\|_H\right],$$

where $m = \inf_{\|x\|=1}(A^p x, x) > 0$, $M = \sup_{\|x\|=1}(A^p x, x) > 0$, *and* $[x]$ *denotes the integer of* x.

In this paper, in contrast to [1] , we will consider a case when instead of exacts initial data $\{A^p; u^p\}$ there are approximately initial data $\left\{A^h, h\ ;\ u^\delta, \delta\right\}$. These data are characterized to the positive data δ and h in the following way: δ characterizes the error of the right-hand side of the equation as $\|u^p - u^\delta\|_H \le \delta$, and h characterizes the error of the operator A^h as $\|A^p - A^h\|_H \le h$. When we have such information about the equation (1) then we could find only approximate solution of the equation (1). Besides this approximate solution converges to the exact solution z^p as δ and h converge to zero independently.

Thus it is necessary to find the approximate solution of the equation

$$A^h z = u^\delta,$$

convergent to the exact solution z^p as $\delta \to 0$ and $h \to 0$ if it is known that

$$\|u^p - u^\delta\|_H \le \delta,$$

$$\|A^p - A^h\|_H \le h,$$

and is known the data $\{\delta; h\}$ also.

Such problem is considered in the works [2; 3; 5]. In these papers are proved the existence theorems of regularizing operators obtaining the way of variation methods: is minimized Tichonoff stabilization functional $\Omega[z]$. In the works [4; 6; 7; 8; 9] are investigated the various questions relating to similar problem.

The following theorem is proved in this work:

THEOREM 3. *Let* z^p *be an exact solution of* (1) *for* $u = u^p \in H$, *i.e.* $A^p z^p = u^p$. *Then the iterative process*

$$\begin{cases} z_{n+1}^{\delta,\ h} = \left(E - \theta \cdot A^h\right) z_n^{\delta,\ h} + \theta \cdot u^\delta, \quad n \ge 0 \\ z_0^{\delta,\ h} = 0 \end{cases} \tag{3}$$

converges to the exact solution z^p *of the equation* (1) *in the norm of given Hilbert space* H , *if the number of iterations* $n = n(\delta, h)$ *is chosen from the conditions*

$$\sqrt{n} \cdot \delta \to 0 \quad as \quad n \to \infty, \delta \to 0 \qquad and \qquad n^2 \cdot h \to 0 \quad as \quad n \to \infty, h \to 0.$$

Moreover, under the condition

$$0 < \theta \le \frac{4}{3 \cdot \|A^p\|_H}$$

the following total error estimate hold for the iterative process (3)

$$\|z^p - z_n^{\delta,\ h}\|_H \le (2\cdot n\cdot m\cdot\theta\cdot e)^{-\frac{1}{2}}\cdot\|z^p\|_H + \left(\tfrac{4}{3}\cdot\tfrac{n}{m}\cdot\theta\right)^{\frac{1}{2}}\cdot\delta +$$

$$+\tfrac{\theta^2}{2}\cdot h\cdot n\cdot(n-1)\cdot\|u^\delta\|_H + O\left(h^2\cdot n^3\right),\quad n\ge 1$$

where $\inf_{\|x\|=1}(A^p x, x) > 0$.

Besides the optimal total error estimate for iterative process (3) *is reached for*

$$n^{optimal} = \left[\sqrt{\frac{3}{8\cdot e}}\cdot\frac{1}{\theta\cdot\delta}\cdot\|z^p\|_H\right] + \left[\frac{4\cdot\theta^2\cdot h + 3\cdot\theta + \sqrt{7\cdot\theta^4\cdot h^2 - 15\cdot\theta^3\cdot + 9\cdot\theta^2}}{3\cdot\theta^2\cdot h}\cdot\|u^\delta\|_H\right],$$

here $[x]$ *denotes the integer of* x .

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