

THE TAYLOR SERIES EXPANSION COEFFICIENTS OF SOLUTIONS OF THE EMDEN - FOWLER TYPE EQUATIONS

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We present the explicit non-recursive formulae for the Taylor series expansion coefficients of the functions $S_n(t)$ defined as a solution of the Emden - Fowler equation $x'' = -nx^{2n-1}$ with the initial conditions $x(0) = 0$, $x'(0) = 1$, where $n = 1, 2, \dots$. Using the Ostrowski - Zabreiko - Lysenko formula [1], [2] one obtains

THEOREM 1. *The nontrivial coefficients of the Taylor series expansion for the function $S_n(t)$ at $t = 0$ are given by $s_1 = 1$ and*

$$s_{2nk+1} = \frac{1}{2nk+1} \sum_{\beta_1+2\beta_2+\dots+k\beta_k=k} (-2)^{-2k+\sum_{i=1}^k \beta_i} (V; \beta_1, \dots, \beta_k, 2nk) \cdot \frac{\prod_{i=1}^k \binom{2i-1}{i-1}^{\beta_i}}{\prod_{i=1}^k (2ni+1)^{\beta_i}}, \quad (1)$$

where $(V; \beta_1, \dots, \beta_k, 2nk) = \frac{(\beta_1 + \beta_2 + \dots + \beta_k + 2nk)!}{\beta_1! \beta_2! \dots \beta_k! (2nk)!}$, $k = 1, 2, \dots$

Corollary 1. The function $S_1(t)$ coincides with the elementary sine $\sin(t)$ with the well known nontrivial coefficients of the Taylor series expansion at $t = 0$: $s_{2k+1} = \frac{(-1)^k}{(2k+1)!}$. Using (1) one obtains the set of identities:

$$\sum_{\beta_1+2\beta_2+\dots+k\beta_k=k} (-2)^{-2k+\sum_{i=1}^k \beta_i} (V; \beta_1, \dots, \beta_k, 2k) \cdot \frac{\prod_{i=1}^k \binom{2i-1}{i-1}^{\beta_i}}{\prod_{i=1}^k (2i+1)^{\beta_i}} = \frac{(-1)^k}{(2k)!} \quad (k = 1, 2, \dots).$$

Corollary 2. The function $S_2(t)$ coincides with the lemniscatic sine $\text{sl}(t)$ [3], which can be expressed through the Jacobian elliptic functions. Hence (1) gives the nontrivial coefficients of the Taylor series expansion at $t = 0$ for the lemniscatic sine $\text{sl}(t) = \text{sn}(t; i)$.

REFERENCES

- [1] A.M. Ostrowki. *Solutions of equations and systems of equations*. Academic Press, New York and London, 1960.
- [2] P.P. Zabreiko and Y.V. Lysenko. Exact formulas for higher order derivatives for inverse functions in banach spaces. *Doklady of the national academy of sciences of Belarus*, **45**, 2001, 27 – 30.
- [3] A. Gritsans and F. Sadyrbaev. Lemniscatic functions in the theory of the Emden - Fowler differential equation. "Mathematics. Differential Equations." *Proc. Inst. Math. Comp. Sci., Univ. of Latvia*, **3**, 2003, 5 – 27.