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STABILITY ANALYSIS ASYMTOTIC METHOD FOR LINEAR DIFFERENCE EQUATIONS WITH MARKOV COEFFICIENTS

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The paper analyzes a behavior of second moments of linear difference equations with coefficients dependent on homogeneous ergodic Markov chain

$$x_t = A\,(y_t)\,x_{t-1}, \quad t \in N \tag{1}$$

where $\{A\,(y)\,, y \in Y\}$ is uniformly bounded continuous $n \times n$ matrix function, $\{y_t, t \in N\}$ is a homogeneous ergodic Feller Markov chain with invariant measure $\mu(dy)$ and transition probability $p(y, dz)$ on the metric compact Y. Let us suppose that matrix $A(y)$ in equation (1) has a form $A\,(y, \varepsilon) := M + \sum\limits_{k=1}^{l} \varepsilon^k A_k\,(y)$ where ε is a small positive parameter and matrix M has spectrum in a following form: $\sigma\,(M) = \sigma_0\,(M) \cup \sigma_\gamma\,(M)$ divided into two parts $\sigma_0\,(M) \subset \{|\lambda| = 1\}$ and $\sigma_\gamma\,(M) \subset \{|\lambda| \leq \gamma < 1\}$. This paper elaborates a convenient for application asymptotic method of stability analysis for equations with near to constant coefficients. The algorithm is based on Laurent series decomposition by small parameter powers of specially constructed quadratic Lyapunov function. Applying the methods and results of [1] this paper proposes an algorithm which allows to reduce the above problem to testing of positive definition property of a solution of the specially constructed matrix equation. To derive mean square stability conditions for (1) we will analyze the spectral properties of linear continuous operator $(Aq)\,(y) := \int\limits_Y A^T\,(z)\,q\,(z)\,A\,(z)\,p\,(y, dz)$ acting in the Banach space V of symmetric uniformly bounded continuous $n \times n$ matrix functions $\{q\,(y)\,, y \in Y\}$ with norm $\|q\| := \sup\limits_{y \in Y, \|x\|=1} |(q\,(y)\,x, x)|$. It is proved that there exists such a positive number ε_0 that for any $\varepsilon \in (0, \varepsilon_0)$ equation (1) with matrix $A(y, \varepsilon)$ is exponentially mean square stable if and only if the equation $A\,(\varepsilon)\,q\,(\varepsilon) - q\,(\varepsilon) = -I$ has solution in a form of Laurent series by powers of ε with positive defined main part $\hat{q}\,(y, \varepsilon) := \sum\limits_{k=-d}^{0} \varepsilon^k q_k\,(y)$, $d \geq 1$. Our approach is based on Kato perturbation theory [2]. The given algorithm is expounded using two examples: exponentially mean square stable difference equation and exponentially mean square unstable difference equation.

REFERENCES

[1] V. Carkova. *Convergence of stochastic iterations.* Latvian University, Riga, 1989. (in Russian)

[2] T. Kato. *Perturbations theory for linear operators.* Springer-Verlag, Berlin-Heidelberg, 1966.