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DIFFUSION APPROXIMATION FOR MARKOV IMPULSE DYNAMICAL SYSTEMS

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The problem of asymptotic analysis of dynamical systems under small random perturbations has been discussed in many mathematical and engineering papers [see [2] and references there]. The approach proposed in [2] makes it possible to apply for asymptotic stochastic analysis of real dynamical systems not only an averaging procedure but also diffusion approximation. It should be noted that in spite of the fact that the above asymptotic methods of stochastic analysis have been developed for differential equations with discontinues right part the phase trajectories have assumed to be continuous random processes. But some dynamical systems of the recent Economics (see, for example, [1] and review there) require an extension of "smooth" models to allow the phase motion to have a jump type discontinuity. As a rule to describe the phase motion $x(t)$ of this system one should define a sequence of random time moments $\{\tau_j^\varepsilon, j \in \mathbf{N}\}$ dividing time axes into small random intervals $\mathbb{E}\{\tau_j^\varepsilon - \tau_{j-1}^\varepsilon\} = 0(\varepsilon)$ of continuity of motion. Here and further ε is small positive parameter. Within an interval $(\tau_{j-1}^\varepsilon < t < \tau_j^\varepsilon)$ a phase trajectory satisfies the differential equation $\frac{dx^\varepsilon}{dt} = f(x^\varepsilon, \xi^\varepsilon(t), \varepsilon)$, but at any moment $t = \tau_{j+1}$ a trajectory endures a jump $x^\varepsilon(t) = x^\varepsilon(t-0) + \varepsilon g(x^\varepsilon(t-0), \xi^\varepsilon(t-0)), \varepsilon)$ with right parts dependent on rapid-changing stochastic process $\{\xi^\varepsilon(t)\}$. The problem arises: how to describe a chaotic behaviour of this dynamical system at a given time interval $[0, T]$ as $\varepsilon \to 0$. As an example of the above mentioned problem one can point out the classical portfolio selection problem of contemporary financial econometrics [1]: at each time moment we would like to construct a portfolio of stocks with rapidly chaotic "choppy" prices in such a way that minimizes an utility (fitting) function which is a functional of stock prices on a given time interval. A possible approach to this problem may be based on the methods and results published in [3]. The proposal paper illustrates this approach by analyzing classical European option pricing problem for trinomial stock market. As it well known this problem has no exact solution [1] and therefore one may find some asymptotic approximation of the desired results. For that we will construct a diffusion approximation of stock price dynamics in order to apply classical Black-Scholes formula. In our model $\xi^\varepsilon(t)$ is stock interest rate simulated as three states homogeneous ergodic Markov process given by infinitesimal matrix $Q(y, z)$, τ_j^ε are switching time moments of this process and $x^\varepsilon(t)$ is stock price at time moment t.

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