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ON ERGODIC GARCH(P,Q) PROCESS

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Over the last decade there has been a tendency [1; 2] to employ for financial time-series data analysis the regression model for conditional mean, defined for endogenous and exogenous variables Y_t and $X_t^{(k)}$ by formula $Y_t = b_0 + \sum_{i=1}^{n} b_i X_t^{(i)} + \xi_t$, $E\{\xi_t/\Phi_{t-1}\} \equiv 0$, $E\{\xi_t^2/\Phi_{t-1}\} \equiv \sigma_t^2$ with errors (shocks) ξ_t given as GARCH(p,q) model that takes the following form:

$$\sigma_t^2 = \theta_0 + \sum_{k=1}^{p} \varphi_k \sigma_{t-k}^2 + \sum_{k=1}^{q} \theta_k \sigma_{t-k}^2 \varepsilon_{t-k}^2, \tag{1}$$

where $\{\varepsilon_t\}$ is a white-noise type time series (that is, i.i.d. random variables with mean zero and variance one) and Φ_{t-1} is a sigma-algebra of information up to time $t-1$, defined by $\{\varepsilon_s, s \le t-1\}$. It is known [2] that under assumption $\sum_{k=1}^{p} \varphi_k + \sum_{k=1}^{p} \theta_k < 1$ there exists a stationary time-series $\{\hat{\sigma}_t^2, t \in Z\}$ defined by (1) and deviations $u_t = \hat{\sigma}_t^2 - \sigma_t^2$ of any other time series σ_t^2 satisfying (1) from $\hat{\sigma}_t^2$ converge to zero with probability one as $t \to \infty$. It should be mentioned that the parameters of regression model (1) are usually estimated by the least square method under condition of stationarity of analyzed time series. For that one needs at least an existence of $E\hat{\sigma}_t^4$ or, as it has been proven in [3], $\lim_{t\to\infty} E|\hat{\sigma}_t^2 - \sigma_t^2|^2 = 0$. This paper proposes the necessary and sufficient condition for exponential decreasing of the above deviations Eu_t^2. For example for GARCH(2,2) this condition has following form:

$$E|\varepsilon_t^2 - 1|^2 < \frac{[(1-\varphi_2-\theta_2)^2 - (\varphi_1+\theta_1)^2](1+\varphi_2+\theta_2)}{(\theta_1^2+\theta_2^2)(1-\varphi_2-\theta_2) + 4(\varphi_1+\theta_1)\theta_1\theta_2}.$$

The final results are given in a very convenient for applications integral form involving parameters of iterative procedure (1). A mathematical background of the results is based on the derived in [4] covariance method for mean square exponential Lyapunov stability analysis of linear difference equations with random coefficients.

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