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ON BOUNDEDNESS OF SOLUTIONS IN CONDUCTIVE-RADIATIVE HEAT TRANSFER MODELS

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We consider a class of conductive-radiative heat transfer models, which can be described by following elliptic boundary value problem:

$$\int_{\Omega}(k_1(\nabla(u+\phi)\cdot\nabla\psi)+k_2(u+\phi)_{x_1}\psi)\,dv+\int_{\Sigma_s}G_1(|u+\phi|^3(u+\phi))\psi\,ds$$

$$=\int_{\Sigma_s}G_2(|\lambda|^3\lambda)\psi\,ds\quad\forall\psi\in\dot{V}_5.$$

Here Ω is bounded cylindrical domain, Σ_s is lateral surface of Ω, Σ_{ht} is heater surface, $u+\phi$ is temperature of Ω and λ is temperature of Σ_{ht}. Linear operators $G_1\colon L_{5/4}(\Sigma_s)\mapsto L_{5/4}(\Sigma_s)$, $G_2\colon L_{5/4}(\Sigma_{ht})\mapsto L_{5/4}(\Sigma_s)$ describe radiative heat propagation within Σ_s, Σ_{ht} system.

The boundedness of weak solutions for the given type of elliptic boundary value problems can be proved in different ways.

The standard approach, that can be used, is to adopt the methods from [1], [2], where general elliptic problems are treated (see [3]). Unfortunately, the estimates for bounds of solutions, which are obtained in this way, are not accurate enough and have complicated dependence from coefficients of the boundary value problem.

Other approach to prove boundedness of weak solutions is to use methods similar with those developed to prove maximum principle of elliptic boundary value problems ([1]). As result accurate estimates for the upper and lower bounds of weak solutions can be obtained:

$$0\le(\phi+u)\le\max\{\|\lambda\|_{L_\infty(\Sigma_{ht})},\|\phi\|_{L_\infty(\partial\Omega\setminus\Sigma_s)}\}.$$

REFERENCES

[1] D. Gilbarg and N.S. Trudinger. *Elliptic partial differential equations of second order.* Springer-Verlag, Berlin, 1983.

[2] O.A. Ladyzenskaja and N.N. Uraltseva. *Linear and quasilinear elliptic equations.* Academic Press, New York-London, 1968.

[3] M. Laitinen and T. Tiihonen. Conductive-radiative heat transfer in grey materials. *Report B 6/2000, University of Jyväskylä,* 2000.