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# ON A METHOD FOR CONSTRUCTION OF SHAPE PRESERVING HISTOSPLINES

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Let $H = \{h_i\}_{i=1}^N$ be a histogram describing a finite sample with the frequency $h_i$ for the class interval $[x_{i-1}, x_i[$, $i = 1, 2, \ldots, N$. We are interested in the construction of shape preserving histosplines, that is splines $s$ satisfying the area-matching conditions

$$\int_{x_{i-1}}^{x_i} s(x)\,dx = h_i, \quad i = 1, 2, \ldots, N,$$

and preserving such essential properties of the histogram as positivity, co-monotonicity and co-convexity.

This problem is solved by quadratic splines $s \in S_2^1(\Delta)$ :

$$S_2^1(\Delta) = \{s \in C^1[t_0, t_{3N}] \,:\, s\,|_{[t_j, t_{j+1}]} \in P_2,\, j = 0, 1, \ldots, 3N-1\}$$

on the net $\Delta = \{t_0, t_1, \ldots, t_{3N}\} \subset [x_0, x_N]$, which is obtained by dividing each interval $[x_{i-1}, x_i[$, $i = 1, 2, \ldots, N$, into three parts with

$$t_{3i-3} = x_{i-1}, \qquad t_{3i-2} = x_{i-1} + \alpha_i(x_i - x_{i-1}),$$
$$t_{3i-1} = x_i - \beta_i(x_i - x_{i-1}), \qquad t_{3i} = x_i, \qquad i = 1, 2, \ldots, N,$$

here $(\alpha_i)_{i=1}^N$, $(\beta_i)_{i=1}^N$ are numbers such that

$$\alpha_i > 0, \quad \beta_i > 0, \quad 1 - \alpha_i - \beta_i > 0, \qquad i = 1, 2, \ldots, N.$$

These splines have two freely chosen additional breakpoints in each class interval $]x_{i-1}, x_i[$ end depend on numbers $\alpha_i$, $\beta_i$, values $y_i = s(x_i)$ and derivatives $m_i = s'(x_i)$, $i = 1, 2, \ldots, N$. We describe necessary and sufficient conditions for the existence of a positive co-monotone and co-convex histospline in terms of regions $DMC_i$, $DYC_i$ in the $m_{i-1}m_i$ and $y_{i-1}y_i$ - planes respectively. The regions $DMC_i$ and $DYC_i$ are investigated as functions of the positions of additional breakpoints. Finally we propose an algorithm for the construction such regions $DMC_i$ and $DYC_i$, $i = 1, 2, \ldots, N$, that provide positive co-monotone and co-convex histopolation with corresponding quadratic splines.