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FRACTAL PECULIARITIES OF THE EQUATIONS OF POPULATION GROWTH

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When exploring real phenomena, the choice of a model is the most significant aspect of applied research. The same real phenomenon may be described from different points of view. This enables to create many abstract conceptions concerning the reality to explore. In order to develop and explore models, different branches of mathematics (mathematical analysis, theory of differential equations, theory of probability etc.) and informatics (object-oriented analysis and designing, imitative modelling, visual programming etc.) are used. The only possible way, how to acquire the modelling, is to demonstrate different phenomena and different models, describing them. Interrelationships among plants and animals and between them and their environment are the significant issue to be explored. The modelling of the number of population is determined by the factors of birth and death.

Maltus' discreet model (1) in the complex plane $\mathbb{C}$ is the formal replacement of real variables, constants and coefficients by corresponding complex variables, constants and coefficients. Then the model may be viewed as iteration in the complex plane, but the results of such process may be described as a fractal portrait

$$\frac{dx}{dt} = \alpha x. \tag{1}$$

In Voltera model the problem becomes two-dimensional. There are two populations in the space - predators and their preys. We obtain the non-linear system of differential equations (2), where α - the increase coefficient of the prey population number, β - the decrease coefficient of the prey population number, when the preys are eaten by the predators, γ - the decrease coefficient of the predators population number if there is a lack of preys

$$\begin{cases} \dfrac{dx}{dt} = x(\alpha - \beta x) \\ \dfrac{dy}{dt} = -y(\gamma - \delta x). \end{cases} \tag{2}$$

Volterra model may be viewed as the one-dimensional reflection of complex plane in relation to oneself. Julia's set is the result of iteration of the complex plane reflections and enables to visualise the behaviour of the non-linear dynamic systems, developing a series of fractal portraits.

REFERENCES

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