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SOME RATIONAL DIFFERENCE EQUATIONS WITH A POSITIVE REAL POWER

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In this talk we consider some second order rational difference equations with a positive real power in the form:

$$x_{n+1} = \frac{\alpha + \beta x_n^k + \gamma x_{n-1}^k}{A + B x_n^k + C x_{n-1}^k}, \quad n = 0, 1, 2, .., \tag{1}$$

with non-negative parameters $\alpha, \beta, \gamma, A, B, C$ and arbitrary non-negative initial conditions x_{-1}, x_0 such that the denominator is always positive, and $k \in (0, \infty)$.

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ON SMOOTHING SPLINES IN CONVEX SETS

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The talk deals with the following conditional minimization problem

$$||Tx||^2 + ||R(Ax - v)||^2 \longrightarrow \min_{x \in B^{-1}(C)},$$

where $T : X \to Y$, $A : X \to I\!R^n$ and $B : X \to Z$ are linear continuous operators in Hilbert spaces X, Y and Z, $R = diag(\sqrt{\rho_i})_{i=1,\dots,n}$ is the diagonal matrix with $\rho_i \geq 0$, $i = 1, \dots, n$, $v \in I\!R^n$ and $C \subset Z$ is a closed convex set. This problem generalizes several approximation problems: the interpolating problem, the smoothing problem with weights, the smoothing problem with obstacles, the problem on splines in convex sets, the mixed interpolating-smoothing problem [1].

We investigate this generalized approximation problem, prove the existence theorem and consider the characterization of its solutions. We show how the theorem gives already known results for the problems mentioned above and obtain new special cases. More detailed analysis is done for some interesting specific problems (see, e.g., [2], [3]).

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LINEAR DIFFERENCE EQUATIONS WHOSE CHARACTERISTIC EQUATION HAS A ROOT -1

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One very often used tool for determining the local stability character of the equilibrium point $\bar{x}$ of a difference equation

$$x_{n+1} = F(x_n, x_{n-1}, ..., x_{n-k}), n = 0, 1, ... \tag{1}$$

is the following result, known as the Linearized Stability Theorem.

Theorem 1. *([1], [2]) Assume that the function F is a continuously differentiable function defined on some open neighbourhood of an equilibrium point $\bar{x}$. Then the following statements are true:*

1. *When all roots of the characteristic equation have absolute value less than one, then the equilibrium point $\bar{x}$ of equation (1) is locally asymptotically stable.*
2. *If at least one root of the characteristic equation has absolute value greater than one, then the equilibrium point $\bar{x}$ of equation (1) is unstable.*

When the characteristic equation has a root 1 or -1 then there is no general methods that can be used to investigate the behaviour of solutions and to determine whether the equilibrium point is locally stable or unstable.

We investigate the behaviour of solutions for linear difference equations

$$x_{n+1} = A_1 x_n + A_2 x_{n-1} + ... + A_{k+1} x_{n-k} + B \tag{2}$$

that have a root -1 of the characteristic equation.

In many cases (not only for linear difference equations but also for some rational difference equations) the root -1 is connected with the period-two solution. The aim of the investigation is to determine why and how the root -1 of the characteristic equation affects the behaviour of solutions.

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TEACHERS' COMPETENCE TO PREPARE STUDENTS FOR MATHEMATICAL OLYMPIADS

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When we talk about the mathematical education in schools we usually mean the regular mathematics classes and curricula of mathematics, but math competitions and Olympiads are an essential part of the general education. For gifted students it is a great way to express themselves, demonstrate their abilities and compare themselves with other pupils. When working with students teacher has to be knowledgeable in the appropriate field. Lately students results in mathematical Olympiads become worse. It can be explained by many factors such as school curricula, students interest, lack of time, lack of optional classes, but one of the most important factors is teachers' competence not only in regular classes mathematics, but also in Olympiad mathematics.

If someone decides to become a teacher of mathematics first decision is to choose an university and a study program. In Latvia there are several options, for example:

- bachelor study program "Teacher of Natural Sciences and Information Technology" (Faculty of Chemistry) at the University of Latvia [1];
- professional master's study program "Teacher" (Faculty of Education, Psychology and Art) at the University of Latvia [2];
- bachelor study program "General Education Teacher" at the Liepaja University [3];
- second level short study program "Secondary Education Teacher" at the Riga Teacher Training and Educational Management Academy [4];
- professional master's study program "Education" at the Daugavpils University [5].

We discuss what courses that are connected with Olympiad mathematics becoming teachers of mathematics study at different universities in Latvia, as well as what opportunities have mathematics teachers to improve and develop their skills at solving mathematical Olympiad problems.

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[4] http://www.rpiva.lv.

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ON STABILITY OF SOLUTIONS OF HILL TYPE EQUATIONS WITH PIECEWISE CONSTANT COEFFICIENTS

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We consider equations of the type

$$x'' + (\delta + Ap(t))x = 0, \tag{1}$$

where $p(t)$ is a function

$$p(t) = \begin{cases} 1, & t \in [0;1], \\ -1, & t \in [1;2], \end{cases} \tag{2}$$

continued to infinity by periodicity. We detect various behaviors of solutions of the equation and show the cases of stability and instability (in the sense that any solution is bounded or there exists an unbounded solution). The related visualizations on a torus are given also.

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PARTIAL FINITELY GENERATED BI-IDEALS

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Blanchet-Sadri et al. in [1] accented that partial words appear naturally in several fields such as DNA computing, data communication, molecular biology etc. Our aim is to aggregate partial words with the class of infinite words that we are interested in, i.e., bi-ideals. Nowadays the importance of information is so expansive that it is not possible to overvalue it, as there are times and reasons for knowing only partial information about something, for example, DNA structure.

As DNA has some certain structure (with possible missing information) and bi-ideals (in this case, finitely generated bi-ideals) have a structure. The idea is to solve the problem of filling the holes (missing information) in finitely generated bi-ideals. In general case of finite amount of holes in finitely generated bi-ideals it is always possible to get the whole information back. Unfortunately, in general case of infinite amount of holes it is not possible.

A *finite partial word* of length n over A is a map $w : \{0, \ldots, n-1\} \to A \cup \{\diamond\}$, where $\diamond \notin A$. The symbol $\diamond$ is viewed as a unknown symbol. A *right infinite partial word* or *infinite partial word* over A is a map $w : \mathbb{N} \to A_\diamond$.

A sequence of finite words $v_0, v_1, \ldots, v_i, \ldots$ is called a *bi-ideal sequence* if for each $i \geq 0$, $v_{i+1} \in v_i A^* v_i$ and $v_0 \neq \lambda$. If $v_0, v_1, \ldots, v_n, \ldots$ is a bi-ideal sequence, then there exists a unique sequence of finite words $u_0, u_1, \ldots, u_n, \ldots$ with $u_0 \neq \lambda$ called the *basis* of the bi-ideal sequence (v_n) such that

$$\begin{aligned} v_0 &= u_0 \\ v_{i+1} &= v_i u_{i+1} v_i. \end{aligned}$$

The infinite word one gets as a limit of this bi-ideal sequence $x = \lim_{n\to\infty} v_n$ is called a *bi-ideal*. The bi-ideal is called *finitely generated* if its basis sequence (u_i) is periodic.

THEOREM 1. *If two irreducible finitely generated bi-ideals x and y are not equal, then there*

$$\overset{\infty}{\exists} i : x_i \neq y_i.$$

THEOREM 2. *It is possible to fill the finite number of holes for a given finitely generated bi-ideal.*

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FRAGMENTARY FUZZY METRICS AND THEIR APPLICATIONS IN THE STUDY OF INFINITE WORDS

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The aim of this talk is to introduce the concept of a fragmentary fuzzy ultrametric, generalizing the concept of a fuzzy ultrametric, to study properties of a fragmentary fuzzy ultrametric and to show that they can be used as a tool for in the study of the set of infinite words, that is more appropriate for this aim than "classical" metrics (see, e.g. [1] or ordinary fuzzy ultrametrics.

A strong fuzzy ultra fragmentary metric m on a set X is mapping $m : X \times X \times (0, +\infty) \to (0, 1]$, satisfying the following conditions for all $x, y, z \in X$, $t \in (0, +\infty)$:
(1FFM) $m(x, y, t) > 0$;
(2FFM) $m(x, y, t) \geq \frac{t}{t+1}$ whenever $x = y$;
(3FFM) $m(x, y, t) = m(y, x, t)$;
(4FFM) $m(x, z, t) \geq m(x, y, t) \wedge m(y, z, t)$;
(5FFM) $m(x, y, \cdot) : (0, +\infty) \to [0, 1]$ is continuous and non-decreasing.
Our definition of a fragmentary fuzzy ultrametric differs from the "standard" definition of a strong fuzzy ultrametric [2] in axiom (2FFM) which we use instead of a more restrictive "standard" axiom (2FM) [2] requesting that $m(x, y, t) = 1$ if and only if $x = y$.

We define a sequence of pseudometrics d_n on the set X of infinite words as follows. Given $x = (x_0, x_1, x_2, \ldots), y = (y_0, y_1, y_2, \ldots) \in X$, let $\chi_i(x, y) = 0$ if $x_i = y_i$ and $\chi_i(x, y) = 1$ if $x_i \neq y_i$. Let $d_0(x, y) = \chi_0(x, y)$, $d_1(x, y) = \chi_0(x, y) + \frac{\chi_1(x,y)}{2}$, $\ldots$ $d_n(x, y) = \sum_{i=0}^{n} \frac{\chi_i(x,y)}{2^i}$, $\ldots$.

Basing on this sequence of ultra pseudometrics we construct the sequence mappings on X: $\mu_0(x, y, t) = \frac{t}{t+1+d_0(x,y)}$, $\mu_1(x, y, t) = \frac{t}{t+1+d_1(x,y)}$, $\ldots$, $\mu_n(x, y, t) = \frac{t}{t+1+d_n(x,y)}$, $\ldots$.

Finally, we define the mapping $\mathfrak{m} : X \times X \times (0, +\infty) \to [0, 1]$

$$\mathfrak{m}(x, y, t) = \begin{cases} \mu_0(x, y, t) & \text{if } 0 < t \leq 1 \\ \mu_1(x, y, t) \vee \mu_0(x, y, 1)(x, y, t) & \text{if } 1 < t \leq 2 \\ \mu_2(x, y, t) \vee \mu_1(x, y, 2) & \text{if } 2 < t \leq 3 \\ \ldots & \\ \mu_n(x, y, t) \vee \mu_{n-1}(x, y, n-1) & \text{if } n < t \leq n+1 \\ \ldots, & \end{cases}$$

show that it is the required fragmentary fuzzy ultra metric and discuss its properties.

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ATTRACTING SETS FOR 2D DIFFERENTIAL SYSTEM ARISING IN GENE REGULATORY NETWORK THEORY

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We consider the differential system

$$\frac{dx_i}{dt} = f\left(\sum W_{ij}x_j - \theta\right) v_g - x_i v_g - \eta, \tag{1}$$

$f(z) = \dfrac{1}{1+e^{-\mu z}}$ is the sigmoidal regulatory function, where $z = \sum W_{ij}x_j - \theta$ and W is a regulatory matrix. Parameter θ is a regulatory parameter which can be adjusted, and μ indicates the gain parameter of the sigmoidal function. We study the attracting sets for various choices of matrix W.

As a sample we mention the system

$$\begin{cases} x_1' = \dfrac{1}{1+e^{-\mu(-x_2-\theta)}} - x_1, \\ x_2' = \dfrac{1}{1+e^{-\mu(\ \ x_1-\theta)}} - x_2, \end{cases} \tag{2}$$

It appears that only critical points of type “stable focus” are possible for the system (2).

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UNIQUENESS OF THE FISHER-RAO METRIC ON THE SPACE OF SMOOTH DENSITIES

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We assume M is an orientable, m-dimensional, compact manifold without boundary. We can identify the space $\mathrm{Dens}(M)$ of smooth densities with the space of m-forms, $\mathrm{Dens}_+(M)$, the space of positive densities, with everywhere positive m-forms and $\mathrm{Prob}(M)$, the space of smooth probability densities, with m-forms with total integral equal to 1. By $\mathrm{Diff}(M)$ we denote the group of diffeomorphisms, i.e., smooth invertible maps $M \to M$ with smooth inverses.

The Fisher-Rao metric is a Riemannian metric on $\mathrm{Prob}(M)$ and is defined as

$$G_\mu^{\mathrm{FR}}(\alpha, \beta) = \int_M \frac{\alpha}{\mu} \frac{\beta}{\mu} \mu \,.$$

It is of importance in the field of information geometry. Restricted to finite-dimensional submanifolds of $\mathrm{Prob}(M)$, so-called statistical manifolds, it is called Fisher's information metric [1]. The Fisher-Rao metric has the property that it is invariant under the action of the diffeomorphism group. The interesting question is whether it is the unique metric possessing this invariance property. A uniqueness result was established in [2] for Fisher's information metric on finite sample spaces. We could show the following, infinite-dimensional analogon, of the uniquess result [3].

Theorem 1. *Let* $\dim M \geq 2$. *Then any smooth Riemannian metric on* $\mathrm{Prob}(M)$*, that is invariant under the action of the diffeomorphism group of* M*, is a multiple of the Fisher-Rao metric.*

In fact we can give are able to classify all $\mathrm{Diff}(M)$-invariant bilinear forms on $\mathrm{Dens}_+(M)$.

Theorem 2. *Let* $\dim M \geq 2$. *Let* G *be a smooth bilinear form on* $\mathrm{Dens}_+(M)$ *that is invariant under the action of* $\mathrm{Diff}(M)$. *Then, for some functions* C_1, C_2 *of the total volume* $\mu(M)$,

$$G_\mu(\alpha, \beta) = C_1(\mu(M)) \int_M \frac{\alpha}{\mu} \frac{\beta}{\mu} \, \mu + C_2(\mu(M)) \int_M \alpha \cdot \int_M \beta \,.$$

This is joint work with Peter W. Michor and Martin Bauer.

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APPROXIMATION WITH POLINOMIAL AND HYPERBOLIC SPLINES FOR CONSERVATIVE AVERAGING METHOD IN MULTILAYERED MEDIA

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After Second World War was very important to raise oil output. In 1970-ties it was essential to make good mathematical models for petroleum and gas exploitation. I together with my colleagues made new type models on the basis of energy conservation for the simpler type problem (one dimensional statement from formulation of three dimensional problem). This step was very important on the basis of weakness of that time computers. We especially worked with thermal methods for petroleum recovering [1]-[11]. For gas exploitation we worked for strong non-homogeneous layer after possible underground nuclear explosion [12]-[14]. The conservative averaging method was developed for two situations. In the first one the integration was made over one sub-domain and for approximation of polynomial type functions [5], [15] were used. In the second situation we had multi-layered media and we introduced polynomial type integral spline [16]-[21]. Now we have introduced hyperbolic type functions and splines [22]. It was done in the case of Cartesian and cylindrical coordinates.

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ABOUT SOME OPEN PROBLEMS OF DIFFERENCE EQUATIONS WITH CONVERGENT SEQUENCE OF COEFFICIENTS[1]

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In books [1] and [2] many open problems and conjectures about second-order and third-order rational difference equations have been formulated. One open problem is connected with investigation of a difference equation if instead of fixed coefficients is considered a convergent sequence.

For example, we consider the equation $x_{n+1} = \frac{\beta x_n}{1+x_{n-1}}$. This equation, called Pielou's equation, was investigated in many articles and books. This is very famous equation because it is a discrete analogue of the delay logistic differential equation that E.Pielou used as a model for the study populations in biology. In [2] is formulated a following problem.

Open Problem 5.24.1. Assume that $\{\beta_n\}$ is a convergent sequence of positive real numbers. Investigate the global character of solutions of the equation

$$x_{n+1} = \frac{\beta_n x_n}{1 + x_{n-1}}, n = 0, 1, ...$$

We give some results about this and similar open problems.

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WANG TILES

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Wang tiles are unit square tiles with colored edges. Hence each tile can be represented as a 4-tuple (N, E, S, W) where N, E, S and W are the colors of the north, east, south and west sides of the square. Tilings with a finite number of prototiles are only considered. In Wang tilings copies of the prototiles are placed at integer lattice points, without rotating or flipping the tiles, so that all tiles are congruent to the given prototiles by translations only. A tiling can then be represented as a function $f : \mathbb{Z}^2 \to \mathfrak{T}$ where $\mathfrak{T} = \{(N_t, E_t, S_t, E_t) \,|\, t \in D\}$ for some finite D. The tiling rule is that in a valid tiling the shared edge between any two tiles that are edge neighbours must have the same color. The tiling problem was introduced by Hao Wang [1].

Problem (Plane Tiling). Given a finite set $\mathfrak{T}$ of Wang tiles, determine whether it admits a valid tiling of the plane.

Wang incorrectly conjectured that every tile set that tiles the plane permits a periodic tiling, that is, has a translational symmetry. Based on this assumption, he gave a general procedure for deciding the tiling problem. His assumption was disproved in 1966 by Berger [2].

Theorem 1. (Berger) *It is undecidable whether or not a finite tile set has a valid Wang tiling for* $\mathbb{Z}^2$.

Kari proved [3] that the tiling problem remains undecidable even if one considers only so-called NW–deterministic tile sets. A tile set $\mathfrak{T}$ is called NW–deterministic if each tile is determined by the north and west colors. That is if $N_s = N_t$ and $W_s = W_t$ then $s = t$ for all $s, t \in D$.

Theorem 2. (Kari) *It is undecidable whether or not a finite NW-deterministic tile set has a valid Wang tiling for* $\mathbb{Z}^2$.

Suprisingly, the argument can be adapted to automaton semigroups [4].

Theorem 3. (Gillibert) *It is undecidable whether or not a given automaton semigroup is finite.*

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IMPULSE TYPE STOCHASTIC DIFFERENTIAL EQUATIONS FOR POPULATION DYNAMICS

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The lecture deals with the modified classical Lotka-Volterra equation for pray-predator dynamics [2]. Our model builds the logistic equation for pray population growth $x(t)$:

$$\frac{dx}{dt} = ax\left(1 - \frac{1}{K}x\right) \tag{1}$$

and the linear equation for predator dynamics which depends on population size $x(t)$:

$$\frac{dy}{dt} = \alpha y + \beta x(t)y \tag{2}$$

In difference on the classical approach to pray-predator competition we assume that intra-specific contacts have an impulse form and become at random time moments $t \in \mathbb{T} := \{\tau_j,\ j \in \mathrm{N}\}$ by formula:

$$x(\tau_j) = x(\tau_j-)[1 + \varepsilon(A(\xi_j) + B(\xi_j)y(\tau_j))] \tag{3}$$

The time moments $\mathbb{T}$ and a random sequence $\{\xi_j, j \in \mathrm{N}\}$ are defined by the ergodic homogeneous compound Poison process with the invariant distribution $\mu(d\xi)$. Applying the diffusion approximation procedure [1] for impulse type dynamical systems and assuming the infinitesimality of time intervals $\Delta_j = O(\varepsilon)$, where ε is a small positive parameter, we derive a differential equation for averaged prey population growth $\bar{x}(t)$ and the Ito stochastic differential equation for normalized deviations $z(t) = (x(t) - \bar{x}(t))/\sqrt{\varepsilon}$.

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ON MATHEMATICAL MODELLING OF PROCESSES IN BIOCHEMICAL REACTORS[1]

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The boundary value problem with suitable boundary conditions which arises in mathematical modelling of enzyme-catalysed biochemical reaction in spherical particle with radius $r \in [0, R]$ for the equation

$$\frac{d^2c}{dr^2} + \frac{2}{r}\frac{dc}{dr} = f(c),$$

where $f(c)$ is the reaction rate at product concentration c, determined by chosen reaction kinetics, was considered in the paper [1]. There was shown that in realizable non Michael-Menten kinetics with $f(c) = \frac{c}{k_1+k_2c+k_3c^2}$ such boundary value problem with appropriate constants k_1, k_2, k_3 could be with multiple solutions. Analogously following up to monograph [3] it was possible to underline multiplicity of steady-state solutions for sensors with homogeneous enzyme layers which are described by equations which corresponds non Michael-Menten kinetics.

Further, using mass balance equation for the substratum in the form

$$\frac{\partial c}{\partial t} + f(c) = -divJ,$$

where J is the mass flux, introducing relaxation time τ_r and taking into account diffusion coefficient D we can modified Fick's law as it was done in [2]

$$J = -D\nabla c - \tau_r \frac{\partial J}{\partial t}.$$

Eliminating from these expressions $\frac{\partial J}{\partial t}$ even in the linear case of function f we obtain partial differential equation of hyperbolic type. This particularly explains experimentally observed oscillatory properties of biochemical reaction processes in sensors

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SOLUTION OF COMPATIBILITY PROBLEM POSED BY A. CIBULIS

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A *polyomino* is a plane figure formed of joining unit squares edge to edge. A classical reference book on polyominoes is [1]. For given polyominoes P_1 and P_2 the *compatibility problem* is to find a polyomino C (common multiple) that can be tiled by copies of P_1 as well as by copies of P_2. Two polyominoes P_1 and P_2 are said to be *compatible* if a common multiple C exists. A *least common multiple* of two compatible figures is a common multiple with the minimum area. Let us note that a common multiple can not necessarily exist. In general this problem is very difficult and algorithm is not known, if any exists. See G. Sichermans website [2] for a vast collection of compatible figures.

Here we focus on compatibility of the domino with polyominoes or to a *viability* of polyominoes [3]. As stated by A. Cibulis and G. Sicherman all n-ominoes up to $n = 12$ are compatible with the domino; moreover, for every n-omino, with $n \leq 12$, eight copies of n-omino suffice. This result has been independently verified by author. The first uncompatible n-ominoes begin with the $n = 13$. Author can confirm that there are only two uncompatible 13-ominoes, besides only seven 13-ominoes require more than eight copies. Number of uncompatible 14-ominoes is not known yet but is less than ten.

The problem (posed by A. Cibulis) on the number M of copies of polyominoes used to construct least common multiple has been solved. Author have proved that this number M can be any integer. Previously only finite number of values of M was known. The proof is constructive and will be discussed in more details here.

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PROBLEM OF THE NUMBER OF POLYOMINO TILINGS AND RECURRENCE RELATIONS

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Let $t(F, P)$ be the number of tilings (solutions) of a given shape F with copies of P.

Problem: for a given polyomino P and integer i find such a polyomino F that $t(F, P) = i$.

This problem is seemed to be very challenging, difficult and at present has been solved only for some simple polyominoes. The stimulus to study the formulated tiling problem was the article of Erich Friedman [1] where one can find the solution for simplest polyominoes, namely dominoes, and information that he managed to show that other rectangles, L tromino, and the L and S tetrominoes also behave this way, but solution for T tetromino is not known. The solution for this remaining and most *difficult* tetromino firstly found (by A. Cibulis) at the end of 2015 will be discussed in more details here. The tiling problem has been positively solved also for the following pentominoes: F, I, L, N, P, U, V, Y. As to a polyomino P having full symmetry there is only one tiling number, i. e. $t(F, P) = 1$.

Some connections of the tiling problem $t(F, P) = i$ with the Fibbonacci [2] and Padovan or Perrin numbers [3], [4] will be discussed. For example, the well-known and almost evident tiling result

$$t(2 \times n, 1 \times 2) = f_n, f_n = f_{n-1} + f_{n-2}, f_1 = 1, f_2 = 2,$$

immediately implies the identity $f_{2n} = f_n^2 + f_{n-1}^2$. Analogously the tiling numbers

$$t(6 \times n, 2 \times 3) = p_n, p_n = p_{n-2} + p_{n-3}, p_1 = 0, p_2 = p_3 = 1,$$

imply the following identity (not mentioned in [3], [4])

$$P_{2n}^2 = P_n^2 + P_{n-1}^2 + 2P_{n-1}P_{n-2},$$

for Padovan or Perrin numbers: $P_n = P_{n-2} + P_{n-3}$, $P_0 = P_1 = P_2 = 1$.

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SOME RECENT RESULTS ON THE LOGICAL ORDER FOR HILBERT SPACE OPERATORS

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The so called *logical*, or *Gudder order* $\preceq$ on the set $\mathcal{S}(H)$ of all self-adjoint operators of a complex Hilbert space H was introduced in [1] and further studied, e.g., in [2,3]. Several characteristics for it are known; for instance,

$$\begin{aligned} A \preceq B \quad &\text{iff} \quad AB = A^2 \\ &\text{iff} \quad A = BP_A \\ &\text{iff} \quad B = A + C \text{ for some } C \perp B, \end{aligned}$$

where P_A is the projection on the closed range $\overline{\mathrm{ran}}A$ of A, and $C \perp B$ means that $BC = 0$.

It turns out that every self-adjoint operator A is completely determined by its restriction to $\overline{\mathrm{ran}}A$. Moreover, if an operator is considered as a set of ordered pairs $(x, A(x))$ with $x \in H$, then

$$A \preceq B \;\text{ iff }\; A|\overline{\mathrm{ran}}A \subseteq B|\overline{\mathrm{ran}}B.$$

On the ground of this equivalence, we obtain simple proofs for various results from [3] characterising the order structure of $\mathcal{S}(H)$, and also for several new ones.

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BLOCK-LENGTH SELECTION FOR THE EMPIRICAL LIKELIHOOD AND THE DEPENDENT BOOTSTRAP

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For weakly dependent observations Kitamura [1] introduced the blockwise empirical likelihood method in the one-sample case. The idea is to construct the blocks of data, then to apply the usual EL method for independent observations.

The implementation of block bootstrap methods for dependent data requires the selection of a block length. A common approach is to choose a block length that minimizes the Mean Squared Error (MSE) function of block bootstrap estimators as a function of the block length. Different data-based methods for the selection of optimal block lengths have been proposed in the literature.

One of the most popular general methods is proposed by Hall, Horowitz and Jing [2] which employs a subsampling method to construct an empirical version of the MSE function and minimizes this to produce an estimator of the optimal block length.

The second general method for selecting the optimal block length is introduced by Lahiri et al. [3]. This method is based on the jackknife-after-bootstrap method and its extension to block bootstrap.

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FIDUCIAL GENERALIZED CONFIDENCE INTERVALS

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In 1989, Tsui and Werrahandi introduced the concept of generalized p-values and generalized variables, which are useful for developing hypothesis tests in the situations where exact tests are not available [1]. In 1993, Weerahandi generalized the concept of a pivotal quantity for a scalar parameter by defining a Generalized Pivotal Quantity (GPQ). Then he proposed a method for constructing confidence intervals based on GPQs – Generalized Confidence Intervals (GCIs) [2].

In 2006, Hannig et al. singled out a subclass of generalized pivotal quantities [3]. They labeled the GPQs in this subclass as Fiducial Generalized Pivotal Quantities (FGPQs). A confidence interval derived from a FGPQ is referred to as a fiducial generalized confidence interval (FGCI). GCIs based on FGPQs are obtainable using the fiducial argument of Fisher (1935) [4] within a suitably chosen framework, such as the structural inference of Fraser (1966) [5]. In fact, Hannig et al. (2006) not only established a clear connection between fiducial intervals and generalized confidence intervals, but also proved the asymptotic frequentist correctness of such intervals [3].

Since the mid 2000s, there has been a revival of interest in modern modifications of fiducial inference. This increase of interest demonstrated itself both the number of different approaches to the problem and the number of researchers working on these problems, and is leading to an increasing number of publications in premier journals. The common thread for these approaches is a definition of inferentially meaningful probability statements about the subsets of the parameter space without the need for subjective prior information [6].

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NONUNIQUENESS OF SEMIDIRECT DECOMPOSITIONS FOR SEMIDIRECT PRODUCTS WITH DIRECTLY DECOMPOSABLE FACTORS AND APPLICATIONS FOR DIHEDRAL GROUPS

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Nonuniqueness of semidirect decompositions of groups is an insufficiently studied question in contrast to direct decompositions. We obtain some results about semidirect decompositions for semidirect products with factors which are nontrivial direct products. We deal with a special case of semidirect product when the twisting homomorphism acts diagonally on a direct product, as well as for the case when the extending group is a direct product. We give applications of these results in the case of generalized dihedral groups and classic dihedral groups D_{2n} - classification of semidirect decompositions and degree bounds for permutation representations.

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VIZUALIZATION OF COMPUTATIONS USING MATRICES

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Design of computational algorithms is an activity which is important for any area of mathematics, both for resarch and teaching purposes. Matrices and linear-algebraic ideas can be used to make algorithms visual, two dimensional and easy to use. We review several known examples of computational algorithms in number theory, linear algebra and polynomial algebra and offer some computational innovations.

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ON COEXISTENCE OF SOLUTIONS OF TWO-POINT BOUNDARY VALUE PROBLEM

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We consider BVP of the form

$$x'' = f(t, x), \quad x(0) = x(1) = 0.$$

Imagine there are two solutions of BVP $u(t)$ and $v(t)$. Along with $u(t)$ and $v(t)$ consider the respective equations of variations

$$y'' = f_x(t, u(t))y, \quad z'' = f_x(t, v(t))z$$

given with the initial conditions

$$y(0) = 0, \quad y'(0) = 1 \quad \text{or} \quad z(0) = 0, \quad z'(0) = 1.$$

If the number of zeros in $(0, 1)$ of $y(t)$ and $z(t)$ is the same, we say, that types of $u(t)$ and $v(t)$ are equal. We show the example of coexistence of two solutions of equal type.

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MANY-VALUED ROUGH SETS INDUCED BY A FUZZY METRIC

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Originating from the fundamental work by Z. Pawlak [1], the theory of rough sets and rough approximations at present has developed into a broad field of mathematics, which has a deep mathematical background as well as many practical applications, specifically in the analysis of imprecise data and in process of dealing with imperfect information. An essential feature of the theory of rough sets is its important and diverse interrelations with fuzzy set theory.

The aim of this talk is to introduce a new type of rough sets on the basis of multi-valued fuzzy relations defined with the help of strong fuzzy metrics.

Recall that a function $m : X \times X \times (0; +\infty) \to (0; 1]$ is called a strong fuzzy metric [2], if it satisfies the following properties:
1) $m(x, y, t) > 0$, $\forall x, y \in X, \forall t \in (0; +\infty)$
2) $m(x, y, t) = 1$ if and only if $x = y$, $\forall x, y \in X, \forall t \in (0; +\infty)$
3) $m(x, y, t) = m(y, x, t)\, \forall x, y \in X, \forall t \in (0; +\infty)$
4) $m(x, z, t) \geq m(x, y, t) * m(y, z, t)$, where $*$ is a t-norm $\forall x, y, z \in X, \forall t \in (0; +\infty)$
5) $m(x, y, \cdot) : (0; +\infty) \to (0; 1]$ is continuous and non-decreasing function
In particular, as the t-norm $*$ one can take the product t-norm $x * y = x \cdot y$, the minimum t-norm $x * y = \min\{x, y\}$ or the Łukasiewicz t-norm $x * y = \max\{x + y - 1, 0\}$ [3].

Applying fuzzy metrics, we suggest a new kind of fuzzy rough approximations. Given a set X we define upper and lower multi-valued fuzzy operators $l : [0, 1]^X \times (0, +\infty) \to [0, 1]^X$ and $u : [0, 1]^X \times (0, +\infty) \to [0, 1]^X$ by setting

$$l(A, t)(x) = \inf_{x' \in X} (m(x, x', t) \to A(x')), u(A, t)(x) = \sup_{x' \in X} (m(x, x', t) * A(x'))\ \forall A \in [0, 1]^X, t \in (0, \infty).$$

We study properties of these approximation operators. In particular, we present some results about the behaviour of these operators in case when the fuzzy metric $m_d(x, y, t)$ is induced by an ordinary metric $d : X \times X \to [0, \infty)$, for example, $m_d(x, y, t) = t/(t + d(x, y))\, \forall x, y \in X,\ t \in (0, +\infty)$ [4].

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COPULA FIT TO THE NONLINEAR PROCESSES IN THE UTILITY INDUSTRY

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Our research studies the construction and estimation of copula-based semi parametric Markov model for the processes which involved in water flows in the hydro plants. The possibility of identifying nonlinear time series using nonparametric estimates of the conditional mean and conditional variance were studied in many papers (see, for example, [1], and references there). As a rule analyzing the dependence structure of stationary time series regressive models defined by invariant marginal distributions and copula functions that capture the temporal dependence of the processes. As it indicated in [1] this permits to separate out the temporal dependence (such as tail dependence) from the marginal behavior (such as fat tails) of a time series. One more advantage of this type regressive approach is a possibility to apply probabilistic limit theorems for transition from deference equations to continuous time stochastic differential equations ([2], [3]). In our paper we also study a class of copula-based semi parametric stationary Markov models in a form of scalar difference equation:

$$t \in Z : X_t = f(X_{t-1}) + g(X_{t-1})\xi_t \tag{1}$$

Regressions (1) are high-usage equations for simulation and parameter estimation of stochastic volatility models [2]. But unfortunately defined by (1) Markov chain has incompact phase space that complicates an application of probabilistic limit theorem. Copula approach helps to simplify asymptotic analysis of (1). In this manner we can make transformation into copula space. But now this equation defines Markov chain on the compact [0, 1]. This makes easier formulate construction for transition probability and further estimators of functions $f(x)$ and $g(x)$. After diffusion approximation in the copula space one can make inverse substitution and derive stochastic differential equation as diffusion approximation for (1).

Dealing with utility company data we have found the best copula describing data - Gumbel copula. As a result constructed equation 1 was used for an imitation of low probability events (in a hydro power industry) and predictions.

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ON DECOMPOSITION OF THE SECOND MOMENT SEMIGROUP FOR MARKOV LINEAR IMPULSE DYNAMICAL SYSTEMS

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Let $x(t) \in \mathbb{R}^d$ be a fast switched right continuous and having a left limit d-dimensional random process satisfying the differential equation

$$\frac{dx}{dt} = A(y(t/\varepsilon), \varepsilon)x \tag{1}$$

at each time interval $\varepsilon\tau_{k-1} \leq t < \varepsilon\tau_k,\ k \in \mathbb{N}$ and having jumps $x(t) = C_0x(t-0)+C(y(\tau_{k-1}), \varepsilon)x(t-0)$ at the time moments $t = \varepsilon\tau_k, k \in \mathbb{N}$. The matrices $A(y, \varepsilon)$ and $C(y, \varepsilon)$ are analytical functions of a small positive parameter ε: $C(y, \varepsilon) = \varepsilon C_1(y) + \varepsilon^2 C_2(y) + \cdots, A(y, \varepsilon) = A_0 + \varepsilon A_1(y) + \varepsilon^2 A_2(y) + \cdots$ and $\det C_0 \neq 0$. Let $\{y(t/\varepsilon), t \in \mathbb{R}\}$ be a homogeneous ergodic compound Poisson type Markov process given on the probability space $\{\Omega, \mathfrak{F}, \mathbf{P}\}$ and with the phase space $\mathbb{Y}$.

The reducibility method and algorithm were derived applying the results of the paper [2], which give possibility to approximate the moments of the solutions of equation (1) by solutions of the special constructed difference equation

$$X_j = G(Y_{j-1}, \xi_j, \varepsilon)X_{j-1}\ ,\ j \in \mathbf{N} \tag{2}$$

where $G(Y_{j-1}, \xi_j, \varepsilon) := C(Y_{j-1}, \varepsilon)\exp\{\varepsilon A(Y_{j-1}, \varepsilon)\xi_j\}$ and $X_j = x(\varepsilon\tau_j), Y_j = y(\tau_j), j \in \mathbb{N}$. It was proved that under some assumption there exists such a basis $\{\mathbf{B}(y, \varepsilon), y \in \mathbb{Y}\}$ in the space of symmetric $d \times d$-matrices that the covariance matrices $q_k = \mathbf{E}\{X_k X_k^T B(y(\tau_k), \varepsilon)\}$ satisfy a linear iterative procedure $q_k = q_{k-1}\Lambda(\varepsilon)$. This allowed to construct the decomposition of the second order moment Lyapunov spectrum for the impulse dynamical system.

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DIRICHLET BOUNDARY VALUE PROBLEM FOR A SYSTEM OF THE SECOND ORDER ASYMPTOTICALLY ASYMMETRIC DIFFERENTIAL EQUATIONS

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We consider systems of the form

$$\begin{cases} x_1'' + g_1(x_1) = h_1(x_1, x_2, \dots, x_n), \\ x_2'' + g_2(x_2) = h_2(x_1, x_2, \dots, x_n), \\ \qquad\qquad \cdots \\ x_n'' + g_n(x_n) = h_n(x_1, x_2, \dots, x_n) \end{cases}$$

along with the boundary conditions

$$x_1(0) = x_2(0) = \cdots = x_n(0) = 0 = x_1(1) = x_2(1) = \cdots = x_n(1)$$

provided that continuous right sides $h_i(x_1, x_2, \dots, x_n)$ $(i = 1, 2, \dots, n)$ are bounded and satisfy the conditions $h_i(0, 0, \dots, 0) = 0$ $(i = 1, 2, \dots, n)$. We suppose that $g_i(x_i)$ $(i = 1, 2, \dots, n)$ asymmetric asymptotically linear functions, therefore at infinity the left sides behave like Fučík equations. We provide the existence results using the vector field rotation theory.

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ON TWO APPROACHES FOR DETERMINING COUNTRIES POTENTIAL ON PURPOSE OF EXPORT OF NON-EXPENSIVE AND LUXURY SERVICES

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The index of the economic attractiveness of a country is determined by evaluation of various economic, financial, social political, technological, etc. criteria, which as a rule are mutually connected and mutually influenced. Each of these criteria involves various non-uniform economic, financial, social, political, legal, educational, scientific innovative, ecological, cultural, etc. factors, each of which can consist of separate and/or indirectly dependent sub-factors hereinafter referred to as "indicators". One of the widely accepted approaches for evaluation of indexes of economic attractiveness in the studied countries is application of the formula $E\left(i,t\right)=\sum\limits_{j=1}^{n}\hat{x}_{i,\,j}\left(t\right),\quad i=\overline{1,m},\ t\in\left[T_{start},T_{end}\right],$ where n stands for number of initial indicators $\left\{x_{i,\,j}\left(t\right)\right\}$ for any of m countries at the moment of time $t\in\left[T_{start},T_{end}\right]$; parameters $\left\{\hat{x}_{i,\,j}\left(t\right),\ t\in\left[T_{start},T_{end}\right]\right\}_{i=\overline{1,\,m}}^{j=\overline{1,\,n}}$ are the result of standardization by some kind of method for the initial indicators $\left\{x_{i,\,j}\left(t\right)\right\}$.

As for our perspective, it is impossible to determine the true economic attractiveness of the studied countries by the presented formula, namely, this formula is certainly impossible to determine what of the economic, financial, social political, etc. indicators in a given period of time exert a significant influence on economic attractiveness of a particular country or a group of countries with "more or less identical" economic, social, demographic, legal, etc. conditions. Moreover, by this formula it is not possible to determine how different are the same indicators in different countries, the economic attractiveness indices of which either notably different to or, conversely, are relatively close. Consequently, unambiguous clusterization of the studied countries according to their genuine economic attractiveness cannot be implemented by the presented formula. Evidently, the foregoing problem will be disappeared, if we will take into account each indicator $\hat{x}_{i,\,j}\left(t\right)$ along with its "individual influence" $w_{i,\,j}\left(t\right)$ (so-called the weight) which, generally speaking, depends both on time t and every specific indicator of each country, i.e. instead of the foregoing formula the following formula is suggested:

$$E\left(i,t\right)=C_1\cdot\sum_{j=1}^{n}w_{i,\,j}\left(t\right)\cdot\hat{x}_{i,\,j}\left(t\right)+C_2,\quad i=\overline{1,m},\ t\in\left[T_{start},T_{end}\right],\tag{1}$$

where constants $C_i \quad (i = 1; 2)$ are chosen in such a way that they be centers of two clusters of the studied countries with the worst and the best values of indicators.

It may seem that with the formula (1) the aforesaid problem disappears. Nevertheless, difficulty is tied to the fact that such "individual influence weights" $w_{i,\,j}\,(t)$ are not a priori known, and may be defined only by the same known indicators $\{\hat{x}_{i,\,j}\,(t)\}$. Consequently, in contrast with the first formula, the right-hand side of the suggested formula (1) consists of the unknown data, so the left-hand side of the formula (1) cannot be calculated. In other words, the formula (1) is an underdetermined system of $m \cdot T_{s\to e}$ linear algebraic equations relative to $m \cdot (n+1) \cdot T_{s\to e}$ sought-for functions $w_{i,\,j}\,(t)$ and $E\,(i,t)$, where through $T_{s\to e}$ is designated the length of the time segment $[T_{start},\, T_{end}]$ in terms of integer standard unit (without loss of generality we assume that this time unit equals to one year).

In the present paper, there are two approaches developed by the authors for investigation of economic attractiveness of 8 post-socialist countries of Central and Eastern Europe: the Republic of Bulgaria, the Republic of Croatia, the Czech Republic, Hungary, Romania, the Republic of Serbia, the Slovak Republic, the Republic of Slovenia. The essence of the first approach is following: by the theory and methods of inverse and ill-posed problems, the system (1) is brought to the widely known problem of finding stable solution for finite-dimensional operator equation of the first kind, which is further solved by the Tikhonov regularization method. The basis of another approach is a rather untraditional idea: initially, the concepts "Degree of favourability of the year", "Degree of succession of the year", "Degree of influence of years' favourability", "Sensitivity switch" are set, so their meanings are a priori unknown and must be defined, and further we address the question of construction of such mathematical model, which would (a) range the studied countries by the set of indices of economic attractiveness by years; (b) define "Degree of favourability of the year" for each year both in terms of each indicator and totally; (c) range the years themselves by "Degree of favourability of the year" and "Degree of succession of the year"; (d) define impacts of "Degree of succession of the year" on the indices of economic attractiveness of the studied countries. As the result, the authors of the present paper able to construct "from scratch" the following mathematical model:

$$x_j = p \cdot \sum_{i=1}^{n} w_i \cdot \hat{x}_{i,\,j}, \;\; \forall j = \overline{1,\, m}; \;\; w_i = w_{\max} - ss \cdot \sum_{j=1}^{m} |\hat{x}_{i,\,j} - x_j|, \;\; \forall i = \overline{1,\, n}, \tag{2}$$

where x_j is the sought-for index of economic attractiveness of j-th $\left(j = \overline{1,\, m}\right)$ country; vector $x = (x_1,\, ...,\, x_m)^T$ is the sought-for ranging of the studied countries by the values of the indicators $\{\hat{x}_{i,\,j}\,(t)\}$ during n years; parameter w_i is the sought-for "Degree of favourability of the year" for i-th $\left(i = \overline{1,\, n}\right)$ year; vector $w = (w_1,\, ...,\, w_n)^T$ is the sought-for "Degree of influence of years' favourability"; parameter $w_{\max} \overset{def}{\equiv} \max\limits_{i=\overline{1,\,n}} \{w_i\}$ is the highest possible evaluation of "Degree of favourability of the year" by all the years: this parameter is also unknown, since $w_i \;\; \left(i = \overline{1,\, n}\right)$ are unknown; parameter $ss \in (0,\, 1]$ is "Sensitivity switch", which stands as sensitivity coefficient of the model (3) to the set of "Degree of succession of the year", i.e. to the residual $\sum\limits_{j=1}^{m} |\hat{x}_{i,\,j} - x_j|$; parameter $p > 0$ is the controllable parameter, so it plays the role of the proportionality coefficient of the sought-for index of economic attractiveness of each of the studied countries to the weighted sum of the indicators of all the studied countries, and this parameter can be chosen arbitrarily, for example, it can be equal with the number of the studied countries, i.e. $p = m$. Also, the investigation of the solvability of the constructed mathematical model (2) and the uniqueness of its solution was also conducted within the frame of the present paper. Moreover, the question of stability of the model (2) to small changes of the initial data was investigated too, and the analytical method for finding the stable solution for the model (2) was suggested. Finally, in the present paper, two computing experiments are conducted – one for each of the described approaches.

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ON THE GINZBURG-FEINBERG PROBLEM OF FREQUENCY ELECTROMAGNETIC SOUNDING FOR UNAMBIGUOUS DETERMINATION OF THE ELECTRON DENSITY IN THE IONOSPHERE

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In the present work, we investigate the following inverse boundary-value problem relative to the sought-for functions $E\,(x,\, z,\, \omega)$ and $V\,(x,\, z)\,,\ (x;\, z,\, \omega) \in [0,\, L] \times \mathbb{R}^1_{++} \times [0,\, \omega_{\max}]$:
– the equation

$$\left\| \nabla_{x,\,z} E\,(x,\, z,\, \omega) \right\|^2_{\mathbb{R}^2} - \varepsilon\,(x,\, z;\, \omega) = 0, x \in (0,\, L)\,,\ \ z \in \mathbb{R}^1_{++},\ \ \omega \in (0,\, \omega_{\max})\,, \tag{1}$$

where $E\,(x,\, z,\, \omega)$ is the electric-field strength; $\varepsilon\,(x,\, z,\, \omega) \overset{def}{\equiv} \frac{m\cdot\left(\omega^2+\nu^2_{\text{effective}}\right)-4\cdot\pi\cdot e^2\cdot(U(z)+V(x,\,z))}{m\cdot\left(\omega^2+\nu^2_{\text{effective}}\right)}$ is the dielectric permeability of the ionosphere; m is the electron mass; ω is the cyclic/angular frequency of the electromagnetic field; e is the elementary electronic charge; $\nu_{\text{effective}}$ is the effective number of collision of an electron with molecules or ions per second, and at each collision an electron on the average passes to a molecule or ion the pulse of the order $m \cdot \vec{r}^2$, where $\vec{r}$ means an ordered velocity imparted to the electron by electromagnetic field $\vec{E}\,(x,\, z,\, \omega)\,;\ N\,(x,\, z) \overset{def}{\equiv} U\,(z) + V\,(x,\, z)\,,\ V\,(x,\, z) \overset{def}{\equiv} \sum\limits_{i=0}^{n} \varphi_i\,(z) \cdot x^i$ is the distribution of the electron density in the ionosphere;
– the Dirichlet boundary conditions

$$\left. E\,(x,\, z,\, \omega) \right|_{(x,\,z)=(0,\,0)} = 0,\ \ \omega \in [0,\, \omega_{\max}]\,; \tag{2}$$

$$\left. E\,(x,\, z,\, \omega) \right|_{(x,\,z)=(L_j,\,0)} = E_j\,(\omega)\,,\ \ \omega \in (0,\, \omega_j) \quad \forall j = \overline{0,\, n}, \tag{3}$$

where the functions $E_j\,(\omega)\,,\ \omega \in (0,\, \omega_j) \quad \forall j = \overline{0,\, n}$ are experimentally measurable functions, and the essence of these values consists in the field phase, i.e. an eikonal (for instance, see [2]) at the measuring points $A_i\,(x = L_i,\, z = 0) \quad \forall i = \overline{0,\, n}$ relative to the phase of some measuring device located at the point $O\,(x = 0,\, z = 0)\,;$
– the additional constraint

$$0 \leqslant |V\,(x,\, z)| \ll U\,(z)\,,\ \ x \in [0,\, L]\,,\ \ z \in \mathbb{R}^1_{+}. \tag{4}$$

In the inverse problem (1)-(4) we suppose that $\omega_{\max} \in \mathbb{R}^1_{++};\ L \in \mathbb{R}^1_{++};\ n \in \mathbb{N};\ L_j \in (0,\, L) \quad \forall j = \overline{0,\, n};\ \omega_j \in (0,\, \omega_{\max}) \quad \forall j = \overline{0,\, n}.$

Let's note that the considered problem arises, generally, at study of the following problems (for instance, see [3]–[9]): propagation of various low-frequency electromagnetic waves in the ionosphere, exosphere and adjacent to its regions of interplanetary space; propagation of radio waves in the ionosphere, i.e. in the upper layers of the Earth's atmosphere; propagation of radio waves of cosmic origin in the solar atmosphere, in the nebulae as well as in the interstellar and interplanetary spaces; propagation of radio waves at laser ranging of the Sun, the Moon and some planets as well as in case of communication with the distant artificial Earth satellites and space rockets; propagation of low-frequency magneto-hydrodynamic and acoustic waves in space environment; propagation of plasma waves both on the ionosphere and the solar corona; propagation of various types of electromagnetic waves in plasma created in vitro (i.e. at the laboratory conditions) at study of gaseous discharge as well as in installations meant for study of controlled thermonuclear reactions, etc.

It is appropriate also to mention here that by now the comprehension and the solid knowledge of the ionosphere have in many respects well-composed and completed character, and therefore, many sections of the discipline about the Earth's atmosphere is unlikely to undergo a change in the future. However, at present there are variety of investigated issues having the corresponding theoretical foundations only in a state of becoming: such fundamental issues as the formation of the ionosphere, the processes in the transition region between the ionosphere in the interplanetary medium, cloud formation mechanisms, wave excitation mechanisms, etc. have not yet been resolved in a satisfactory extent, and even there are variety of gaps and contradictions in the theories constructed for them.

In the present work, it is proved the existence and uniqueness of the solution of the inverse problem (1)-(4) as well as it is proposed the analytical method (see [10]) permitting to reduce this problem, at first, to the standard problem of integral geometry and, thereupon, to the first kind matrix integral equation of Volterra type with a weak singularity (for instance, see [11]).

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THE ALGORITHM FOR CONDITIONAL STOCHASTIC OPTIMIZATION PROBLEM SOLVING

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Financial mathematics and investment theory are relates with the certain class of questions, which from mathematical point of view can be formulated as conditional stochastic optimization problems. Problem is conditional because there are restrictions on variables similar to those which we have in Lagrange multipliers method for maximum with constrains. Problem is stochastic because there is Wiener noise added to smooth function. In modelling tasks this noise is modelled with random number generator, which add random values to given smooth function. Problem is related to optimization because it is necessary to find maximum or minimum for sum of smooth function and noise with constrained variables. The additional difficulty is related with the high dimensionality of problem in practically interesting cases the amount of variables is several thousands. We consider several methods of large scale systems of linear and non-linear equations solving, with amount of equations about 10000. We compare those methods and suggest our own method based on multi-dimensional thetraedron motion. We describe the received results and work out recommendations for our method implementation for conditional stochastic optimization problems solving. Practical examples of method implementation are considered.

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NEYMAN SMOOTH TEST FOR DEPENDENT DATA IN CASE OF COMPOSITE HYPOTHESIS

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Neyman's smooth goodness-of-fit test is used to verify the following null hypothesis

$$H_0 : f(x) \in \{f(x;\beta\}, \beta \in B\},$$

where $B \subset \mathbb{R}^q$ against the alternative hypothesis

$$H_1 : f(x) \notin \{f(x;\beta\}, \beta \in B\},$$

where $\{f(x;\beta\}, \beta \in B\}$ is a given family of densities (for example, the family of exponential or normal densities) [1].

Ledwina et al. [2] proposed a new version of the Neyman's smooth test for independent data in case of composite hypothesis. It is shown that this test is consistent at essentially any alternative [3].

Munk et al. [4] proposed a modification of the Neyman's test in case of simple hypothesis for general α-mixing processes. Some well known classes of processes, such as ARMA, GARCH, many Markov chain processes possess α-mixing property. In this work a modification of Neyman's test has been proposed for composite hypothesis. The consistency of the proposed test is shown using Monte Carlo simulations.

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SOME ASPECT FOR USING THE CONSERVATION AVERAGING METHOD

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We consider the conservation averaging method (CAM) for solving the 3-D boundary-value problem of second order in multilayer domain. CAM by using integral parabolic type splines was developed by A. Buikis in his Doctoral Thesis [2] and in [3].

The special integral hyperbolic and exponential type splines, with middle integral values of piece-wise smooth functions interpolation are developed [1]. With the help of these splines the problems of mathematical physics in 3-D multilayered domain (also for axis-symetrical domain) with piece-wise coefficients are reduced with respect to one coordinate to 2-D problems. Each layer of these splines contain parameters, which can be selected in order to decrease the error of the solution. These splines are obtained from the general spline with two fixed functions. The parameters of these functions are the characteristic values for the corresponding homogeneous ODEs of second order in fixed direction. These parameters are the best parameters for minimal error. If the parameters tend to zero, we obtain the integral parabolic spline.

This procedure also allows reduce the 2-D problems to 1-D problems and the solution of the approximated problems can be obtained analytically. We chose the CAM for engineering calculation.

In the case of constant piece-wise coefficients we obtain the exact discrete approximation of a steady-state 1-D boundary-value problem.

The solution of corresponding averaged 3-D initial-boundary value problem is also obtained numerically using the discretization in space with the central differences. The approximation of the 3-D non-stationary problem is based on the implicit finite-difference and the implicit alternating method by Douglas and Rachford (ADI) [4]. The numerical solution is compared to the analytical solution [1].

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MATHEMATICAL MODELLING OF MHD FLOW IN THE COMBUSTION PROCESS

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In this paper we discuss the influences of an external magnetic field and the swirl number on the characteristics of flame in the combustion chamber. We consider a simplified model reckoning the interaction of swirl flow and the MHD effects due to the Lorentz force acting on the weakly ionised gas. We present the numerical simulation results for the viscous, incompressible, laminar, axisymmetric swirling flow in a cylindrical pipe with axial uniform magnetic field. The modelling of the combustion process is governed by a single step exothermic chemical reaction of fuel and oxidant. The rate of the reaction is given by one-step first-order Arrhenius kinetics.
We continue works of [1],[2] and consider the following equations:

$$\begin{cases} \frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial x} = 0, \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial x} - \Gamma^2\frac{v^2}{r} = -\frac{\partial p}{\partial r} - Su + \frac{1}{Re}\Delta' u, \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial x} + \frac{u\,v}{r} = \frac{1}{Re}\Delta' v, \\ \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial x} = -\frac{\partial p}{\partial x} - \frac{Gr}{Re^2}T + \frac{1}{Re}\Delta w \end{cases} \tag{1}$$

and

$$\begin{cases} \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial x} = P_1\triangle T + \beta_1 AC_1 e^{\frac{-\delta_1}{T}} + \beta_2 AC_2 e^{\frac{-\delta_2}{T}}, \\ \frac{\partial C_1}{\partial t} + u\frac{\partial C_1}{\partial r} + w\frac{\partial C_1}{\partial x} = P_2\triangle C_1 - AC_1 e^{\frac{-\delta_1}{T}}, \\ \frac{\partial C_2}{\partial t} + u\frac{\partial C_2}{\partial r} + w\frac{\partial C_2}{\partial x} = P_2\triangle C_2 - AC_2 e^{\frac{-\delta_2}{T}}, \end{cases} \tag{2}$$

where $P_1 = \frac{Le}{Pe}, P_2 = \frac{1}{Pe}$; Pe and Le are Péclet and Lewis numbers, Γ is the swirl number, $\beta_{1,2}$ are the heat release parameters, $\delta_{1,2}$ - the scaled activation energies; Re, S and Gr are respectively Reynolds, Stewart and Grashof numbers.
Fields of stream function, vorticity, temperature, and fuel concentration in the cylindrical pipe are obtained for various values of parameters in the model.[1]

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ON MATHEMATICAL MODELLING OF THE COMBUSTION PROCESS OF BIOMASS

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Combustion or burning is a high-temperature exothermic chemical reaction of fuel and an oxidant producing flame and the heat . With the aim to develop a cleaner and more effective heat energy production using the different types of biomass, the research group from LU and RTU proposes to provide a complex research of the processes of combustion and heat energy production using swirling flows for process control. A set of control parameters determining stability and efficiency of these processes at thermo-chemical conversion of the biomass essential for mathematical modelling and numerical simulation are to be obtained as the result of experimental investigation.

The experimental study and mathematical modelling of the magnetic and electric fields in the combustion dynamics at biomass thermochemical conversion are carried out with the aim to provide control of the processes developing in swirling flame reaction zone [2].

Mathematical model is developed taking into the account the axial and azymuthal components of flow velocity and development of the exothermic reaction of fuel combustion downstream the cylindrical combustor [1], [3].

The flame flow formation with the axial velocity at the inlet of the combustor is affected by the axially symmetric magnetic field. The magnetic field is induced by a direct electric current in a coil, which is placed at the inlet of the combustor. The distribution of a stream function, azymuthal component of velocity, vorticity and the formation of the velocity and temperature profiles are calculated for different values of electrodynamic force parameter and swirl number.

The results of numerical simulation have shown that the increase in electrodynamic force parameter couses the magnetic field-enhanced increase of maximal flow velocity, whereas the increase in flow vorticity results in decreasing the peak flame temperature and the rate of the reaction increasing flow vorticity, whereas decreasing peak flame temperature and the rate of reaction [1].

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ON AN EXAMPLE OF THE SECOND ORDER DIFFERENTIAL EQUATION WITH CUBIC NONLINEARITY DEGENERATE ON AN SUBINTERVAL

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The boundary value problem

$$x'' = -ax + \beta(t)x^3, \quad a > 0, \tag{1}$$

$$x(0) = 0, \quad x(1) = 0 \tag{2}$$

is considered, where $\beta(t)$ is a step-wise function of the form

$$\beta(t) = \begin{cases} b, & t \in [0, \delta), \\ 0, & t \in [\delta, 1-\delta], \\ b, & t \in (1-\delta, 1], \end{cases} \tag{3}$$

and $b = 200$, $\delta = 0.45$.

We compare the number $N(a) + m$ of solutions of the problem (1), (2) with the number $N(a)$ of solutions of boundary value problem for the equation $x'' = -ax + bx^3$.

We explain why $N(a) + m > N(a)$.

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DISCRETE HIGHER DEGREE F-TRANSFORM BASED ON B-SPLINES

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The concept of the fuzzy transform (or F-transform) was introduced by I. Perfilieva [3]. The ordinary F-transform can be generalized [4] to F-transform with m-degree polynomial components (the F^m-transform), $m \geq 0$. Fuzzy transforms have applications in image processing, data analysis and time series analysis. Another generalization of the fuzzy transform, the discrete F^m-transform, is particularly important in applications.

It has been shown [2] that the F^m-transform w.r.t. a generalized uniform fuzzy partition given by B-splines of degree $2k-1$ have desirable approximation properties; it was proved that in this case the inverse F^m-transform is exact for polynomials of degree at most $2m+1$, provided that $m \leq k-1$. However, this was done in the setting of the continuous F^m-transform. In practice the available data often is discrete and the continuous approach cannot be applied.

We consider the discrete F^m-transform w.r.t. a generalized uniform fuzzy partition given by B-splines of degree $2k-1$, i.e., the discrete version of its continuous counterpart. In our setting, the discrete function is defined in a uniform grid and the nodes of the fuzzy partition form its sub-grid. It is assumed that the data is generated by some underlying function f. The generalized uniform fuzzy partition is generated by B-splines of degree $2k-1$ and the discrete F^m-transform is defined analogously to the F^m-transform of type I in [1]; we assume that $m \leq k-1$.

We show that the discrete inverse F^m-transform exhibits similar approximation properties as its continuous version, i.e., when f is a polynomial of degree at most $2m+1$, the discrete inverse F^m-transform coincides with this polynomial on the interval where Ruspini condition is fulfilled. Notice that in general the discrete inverse F^m-transform coincides with the polynomial only when its degree is at most m. Hence, B-splines can be used also in the discrete case to improve approximation properties of the inverse F^m-transform.

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INFLUENCE OF INTERNAL HEAT SOURCES ON CONVECTIVE STABILITY OF FLOWS IN A CYLINDRICAL DOMAIN

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Consider an infinitely long circular cylinder filled with a viscous incompressible fluid. A steady convective motion in the cylinder is caused by internal heat sources distributed within the fluid. The problem is described by the system of the Navies-Stokes equations under the Boussinesq approximation. We consider the following two cases: (a) heat sources of a uniform density (as a result of Joule heating) and (b) heat sources distributed in accordance with the Arrhenius' law in a chemically reacting fluid. The base flow in case (a) is found analytically while in case (b) it is obtained numerically using Matlab routine bvp4c.

Linear stability problem for both cases is solved by the pseudospectral collocation method based on Chebyshev polynomials. Results of numerical computations for case (a) are compared with experimental data. The critical Grasshof numbers obtained numerically agree well with experimental data. The most unstable mode is the mode with the azimuthal wavenumber $n = 1$.

The second mode of instability is found in both cases as the Prandtl number grows. This type of instability is associated with thermal running waves. It is found that the second mode in case (b) appears for smaller Prandtl numbers.

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THE VARIETIES OF RICKART RINGS AND REDUCED RICKART RINGS

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T.P. Speed and M.W. Evans showed in [1] that the class of commutative Rickart rings is a variety. The author proved the same for the class of right Rickart rings and for the class of reduced Rickart rings.

Let us recall some well-known definitions.

A ring $\langle R, +, \cdot \rangle$ is called a *right Rickart ring* if for each $a \in R$ there is some idempotent $e \in R$ such that for all $x \in R$,

$$ax = 0 \Longleftrightarrow ex = x.$$

Dually, a ring $\langle R, +, \cdot \rangle$ is called a *left Rickart ring* if for each $a \in R$ there is some idempotent $e \in R$ such that $xa = 0$ iff $xe = 0$ for all $x \in R$.

A ring $\langle R, +, \cdot \rangle$ is called a *Rickart ring* if it is both a right Rickart ring and a left Rickart ring.

Furthermore, a ring $\langle R, +, \cdot \rangle$ is called *reduced* if it has no non-zero nilpotent elements (i.e., if $a^n = 0$, for some natural number n and some $a \in R$, then $a = 0$).

The following theorem describes the equational classe of right Rickart rings.

Theorem 1. *A ring with unity $\langle R, +, \cdot, 1 \rangle$ is a right Rickart ring if and only if there is a unary operation $'$ satisfying the following axioms:*

1. $aa' = 0$,
2. $a'' = 1 - a'$,
3. $(a''b)'' = (a''b)''\,(ab)''$.

Hence, the class of right Rickart rings is a variety. Of course, a similar theorem holds for left Rickart rings.

Reduced Rickart rings also form an equational class (and thus a variety), as the following result demonstrates:

Theorem 2. *A ring with unity $\langle R, +, \cdot, 1 \rangle$ is a reduced Rickart ring if and only if there is a unary operation $'$ satisfying the following axioms:*

1. $aa' = 0 = a'a$,
2. $aa'' = a$,
3. $(ab)' = a' + b' - a'b'$.

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THE STOCHASTIC STABILITY OF A PIPELINE AFFECTED BY PULSATIVE FLUID FLOW

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The deflection $u(t,z)$ of the center line of a pipeline is described by the equation

$$EJ\frac{\partial^4 u}{\partial x^4} - P(t)\frac{\partial^2 u}{\partial x^2} + D\frac{\partial u}{\partial t} + m\frac{\partial^2 u}{\partial t^2} = 0 \tag{1}$$

assuming boundary conditions in a form of equalities $u(t,0) = u(t,L) = 0$; $\frac{\partial^2 u(t,0)}{\partial x^2} = \frac{\partial^2 u(t,L)}{\partial x^2}$. This equation is used by many authors (see, for example, [1]) for analysis of the transverse oscillations of a pipeline's section of length L under influence of a pulsed fluid flow. The coefficients in equation (1) have the following meaning: $P(t)$ – the periodic longitudinal force, EJ – flexural rigidity of a pipeline, m – mass of a pipeline of unit length, D – a dissipation factor. The authors of paper [1] decompose solution of equation (1) in Fourier series $u(t,x) = \sum_{n=1}^{\infty} T_n(t)\sin\left(\frac{n\pi x}{L}\right)$.

Under assumption of sufficiently large mass and infinitesimally small dissipation factor the equation (1) can be rewritten with small positive parameter ε:

$$\frac{d^2x}{dt^2} + 2\varepsilon^2\delta\,\frac{dx}{dt} + \left(\omega^2 + \varepsilon h(y(t))\right)x = 0, \tag{2}$$

where $y(t)$ is Poisson process with infinitesimal operator $Qv(y) = \lambda\int_0^1 [v(z) - v(y)]dz$. Intervals between jumps have exponential distribution with intensity λ and values of jumps ξ_n are i.i.d. $U(0;1)$. Using the second Lyapunov method the stability condition in the form (3) is derived

$$\delta > \frac{\lambda}{4(\lambda^2 + 4\omega^2)}\int_0^1 h^2(y)dy. \tag{3}$$

The diffusion approximation of the equation (2) has the form $dr(t) = a(r)dt + \sigma(r)dw(t)$, where

$$\begin{aligned} a(r) &= \frac{h^2(y)}{8\omega^2}\frac{\lambda}{\lambda^2 + 4\omega^2}(2r-1) - r\delta, \\ \sigma(r) &= \frac{h(y)}{2\omega}\sqrt{\frac{\lambda r}{\lambda^2 + 4\omega^2}}. \end{aligned}$$

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SENSITIVITY ANALYSIS OF MATRIX GAMES

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We consider a zero-sum Matching Pennies game where two players (labelled Player 1 and Player 2) each have a coin that for each trial shows either a head or a tail (description of the game from [2]). If both players show a head, then the second player (Player 2) wins 3 monetary units. If both players show a tail, then Player 2 wins 1 monetary unit. If one player shows a head and the other shows a tail, then Player 1 wins 2 monetary units. According to the above-mentioned description, we can model this game by the following payoff matrix:

		Player 2	
		H	T
Player 1	H	$(-3;3)$	$(2;-2)$
	T	$(2;-2)$	$(-1;1)$

Each zero-sum matrix game can be formulated as a linear programming problem for which we need to find an optimal strategy. In [2] after representing the game as a linear programming model, the sensitivity analysis of elements of the payoff matrix is presented. The game value and the optimal strategies for different values of parameters are determined and compared. Similar analysis was performed in [1].

We generalize this method of sensitivity analysis for two players $2 \times n$ matrix games.

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THE CONDITIONS FOR SOLVABILITY OF RESONANT PROBLEMS

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We consider the problems

$$x^{(4)} - k^4 x = q(t), \tag{1}$$

$$x(0) = x''(0) = x(1) = x''(1) = 0 \tag{2}$$

and

$$x^{(4)} - k^4 x = q(t), \tag{3}$$

$$x(0) = x'(0) = x(1) = x'(1) = 0, \tag{4}$$

provided that the homogeneous problems

$$x^{(4)} - k^4 x = 0, \tag{5}$$

$$x(0) = x''(0) = x(1) = x''(1) = 0 \tag{6}$$

and

$$x^{(4)} - k^4 x = 0, \tag{7}$$

$$x(0) = x'(0) = x(1) = x'(1) = 0, \tag{8}$$

have non-trivial solutions. We obtain the conditions on a function $q(t)$ in order the problems (1),(2) and (3),(4) to be solvable.

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AGGREGATION OF FUZZY VALUES BASED ON EQUIVALENCES

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The talk is devoted to the special constructions of general aggregation operators, which are based on an equivalence relation. The need for such operators acting on fuzzy structures arises dealing with particular decision making problems, multi-objective optimization and other problems, where it is important to take into account an equivalence relation between the objects of aggregation.

Initially, the general aggregation operator based on an equivalence relation has appeared in our research on a choice of optimal solutions for bilevel linear programming problems [1]. Later we have generalized this concept by involving a fuzzy equivalence relation instead of a crisp one (see, e.g., [2]). As a result, we obtained upper and lower general aggregation operators based on fuzzy equivalences. This allowed us to propose the tool, which provides upper and lower approximations of the pointwise and t-norm extension of an ordinary aggregation operator [3].

We investigate properties of these constructions of general aggregation operators (see, e.g., [4], [5]) and show their advantages in comparison with the classical aggregation methods. The main attention in this talk will be paid to aggregation of fuzzy numbers and possible applications of the proposed aggregation technique in decision making.

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Acta Societatis Mathematicae Latviensis
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ON THE ASYMPTOTIC TOPOLOGY OF GROUPS AND MANIFOLDS

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In this talk, I will present a review of some old and recent research on the topology at infinity of universal covering spaces of closed aspherical (3)-manifolds. In particular, I will introduce some results concerning the well-known *Universal Covering Conjecture*, and a few related problems on the *asymptotic topology of discrete groups.*

The Universal Covering Conjecture (which affirms that the universal covering space of a closed, orientable, aspherical 3-manifold is homeomorphic to the Euclidian 3-space) was actually one of the main problems in 3-dimensional topology, after Poincaré Conjecture, although they are both proved nowadays thanks to Perelman's results. To better understand the topological behavior at infinity of such spaces, I will introduce the main topological tameness condition at infinity, namely the *simple connectivity at infinity* (which generalizes the condition that complements of large compacts are simply connected). The main application of this condition is for 'detecting' Euclidean spaces among open contractible manifolds, but it can also be defined for finitely presented discrete groups.

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MODELING THE INFLUENCE OF MAGNETIC FIELDS ON OXYGEN DISTRIBUTION

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The influence of external magnetical fields on a flame is well known ([1], [2]). A part of this effect is attributed to the magnetohydrodynamical effects due to Lorentz force acting on the weakly ionised gas, and another part on the fact that oxygen is paramagnetic (i.e., it is attracted by external magnetic fields) whereas most other gases are diamagnetic, hence magnetic field influences the distribution of oxygen [3] and the whole combustion process.

The effects of thermal convection are reduced when the flame is considered in microgravity [4]. Then the magnetic forces become dominant, however, they only act on the paramagnetic species, namely, the oxygen. We consider the advection-diffusion-reaction system, where the single chemical reaction is modeled by a single step Arrhenius law. The results of a numerical study of temperature and the distribution of oxygen in a gas mixture are presented. Approximation of the nonlinear system is based on the finite difference method.

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DYNAMIC PROGRAMMING APPROACH FOR THE FLOW SHOP PROBLEM

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In this paper, a permutation flow shop scheduling problem (PFS) with a makespan criterion is inspected. In a PFS, n jobs have to be processed, in the same order, on each of m machines. The processing time for each operation $o_{i,k}$ associated with a specific machine $k \in K = \{1, \ldots, m\}$ and a specific job $i \in I = \{1, \ldots, n\}$ is $p_{i,k}$. The objective is to find a permutation of n jobs such that the time at which all the jobs are completed is minimal. The processing order of the jobs is identical for all jobs and starts with machine 1 and ends with machine m.

Bautista et. al. [1] introduced the bounded dynamic programming (BDP) approach for obtaining an exact solution of PFS. BDP method combines features of dynamic programming with features of branch and bound method. In the current work, this method is further developed and the computational results outperforms those given by Companys et. al. [2]. Moreover, several benchmark instances are solved which have been open for some time.

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VALIDATION OF TWO-SAMPLE LOCATION-SCALE MODEL

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In medicine quite often arise a problem to determine the effectiveness of a new drug or treatment. One of the possible ways is to test whether two groups of patients belong to location-scale family. Suppose that we have two samples $X_1, \ldots, X_n \sim F_1$ and $Y_1, \ldots, Y_m \sim F_2$, usually called control and treatment responses. These two samples follow the classical location-scale model if the relationship

$$F_1(t) = F_2\left(\frac{t-\mu}{\sigma}\right), \quad t \in \mathbb{R}$$

holds with some constants $\mu \in \mathbb{R}$ and $\sigma > 0$.

There has been derived several methods to check for the location-scale model. Doksum and Sievers [2] defined general shift function, Valeinis, Cers and Cielens [1] introduced empirical process approach based on quantile-quantile and probability-probability plots, Valeinis also studied confidence bands via empirical likelihood method for sample quantile difference function in his PhD thesis (2007).

Though if we need to estimate the location and scale parameters we face the plug-in empirical likelihood method which was first introduced by Hjort, McKeague and Van Keilegom [3] for one-sample case. According to their setup we derive limiting distribution for the two-sample case. Hence we aim to compare previous mentioned methods to plug-in empirical likelihood method when testing the location-scale model. We present some simulation study as well as practical data examples.

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REDUCIBILITY BY MOMENTS OF LINEAR IMPULSE MARKOV DYNAMICAL SYSTEMS WITH ALMOST CONSTANT COEFFICIENTS.

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This paper deals with linear impulse dynamical systems with phase coordinates $x(t) \in \mathbb{R}^d$ for $t \geq 0$ has been defined on probability space $(\Omega, \mathcal{F}, \mathcal{F}^t, \mathbf{P})$ by the following formulas:
• initial condition

$$x(0) = x \tag{1}$$

• differential equation:

$$\frac{dx}{dt} = A(t, y(t/\varepsilon), \varepsilon)\, x \tag{2}$$

on any interval $(\varepsilon\tau_{j-1} < \ t \ < \varepsilon\tau_j), \quad j \in \mathbb{N};$
• condition of jumps:

$$x(\varepsilon\tau_j) = G(\varepsilon\tau_j, y(\tau_{j-1}), \varepsilon) x(\varepsilon\tau_{j-1}) \tag{3}$$

at time moments $t \in \{\varepsilon\tau_j, \quad j \in \mathbb{N}\}$.

This impulse dynamical systems parameters depend on ergodic piece-wise constant Markov process with values from some phase space $\mathbb{Y}$ and on small parameter ε. Trajectories of Markov process satisfy a system of linear differential equations with close to constant coefficients on its continuity intervals, while its phase coordinate changes discontinuously when Markov process's switching occur. Jump sizes depend linearly on the phase coordinate and are proportional to the small parameter ε. We propose a method and algorithm for choosing the base $\mathbb{B}\,(t, y)$ of the space $\mathbb{R}^d$ that provides approximation of average phase trajectories by a solution of a system of linear differential equations with constant coefficients.

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ESTIMATES FOR SOLUTIONS OF LINEAR ODE[1]

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Let $\dot{x} = A(t)x$ be a linear differential equation with fundamental matrix $X(t,s)$, where $X(s,s) = I$ and I is identity matrix. For proving the centre manifold theorem, reduction principles etc. [1] we use inequality

$$\sup_s \int_s^{+\infty} |X(t,s)|\, dt = \mu < +\infty.$$

In this case it is possible for $t \geq s$ prove the integral exponential estimate

$$\int_t^{+\infty} |X(\tau,s)|\, d\tau \leq \mu \exp\left(-\frac{1}{\mu}(t-s)\right).$$

We construct the scalar differential equation whose solution on the one hand has infinite Lyapunov exponent [2], on the other hand has integral exponential estimate

$$\dot{x} = \left(\frac{a'(t)}{a(t)} - a(t)\right)x, \quad a(t) = a + \eta(t), \quad a > 0, \quad \eta(t) \geq 0,$$

where the function $\eta\colon \mathbb{R} \to \mathbb{R}$ is sawtooth piecewise linear and satisfies the estimates

$$\int_{-\infty}^{+\infty} \eta(t)\, dt < +\infty, \qquad \lim_{n\to+\infty} \frac{\ln(\eta(n))}{n} = +\infty.$$

Then the solution of equation is

$$x(t,s) = \exp\left(-\int_s^t a(\tau)\, d\tau\right)\frac{a(t)}{a(s)} \text{ and } \int_s^{+\infty} x(t,s)\, dt = \frac{1}{a(s)} \leq \frac{1}{a},$$

$$\limsup_{t\to+\infty} \frac{\ln(x(t,s))}{t} = -a + \lim_{n\to+\infty} \frac{\ln(\eta(n))}{n} = +\infty.$$

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ON SOLUTIONS OF HAMILTONIAN SYSTEMS

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We consider Hamiltonian systems of the form

$$\begin{cases} x' = \dfrac{\partial H}{\partial y}, \\ y' = -\dfrac{\partial H}{\partial x}. \end{cases} \tag{1}$$

We study the related boundary value problems with the conditions

$$x(0) = x(1) = 0, \tag{2}$$

$$x(0) = y(1) = 0, \tag{3}$$

$$y(0) = y(1) = 0. \tag{4}$$

The estimates of the number of solutions of boundary value problems are obtained. Various cases of behaviour of solutions are analyzed.

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THE ROLE OF THE FUČÍK SPECTRUM FOR SOLVABILITY OF BOUNDARY VALUE PROBLEMS

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For classical BVP of the type

$$x'' + \lambda x = h(t), \;\; x(0) = 0, \; x(1) = 0,$$

the following statement is known: if λ is an eigenvalue of the homogeneous problem then the problem may be unsolvable for some specific $h(t)$, in contrast to the case of λ not being an eigenvalue. In this case the problem is solvable for any $h(t)$.

For more general problem

$$x'' + \lambda x = h(t, x, x'), \;\; x(0) = 0, \; x(1) = 0,$$

if λ is not an eigenvalue the problem is solvable for any bounded continuous $h(t, x, x')$. Otherwise the existence is not quaranteed even for bounded h.

In case of the Fučík type equation

$$x'' + \mu x^+ - \lambda x^- = h(t, x, x') \tag{1}$$

with corresponding boundary value conditions the two-dimensional spectrum has to be analyzed. The plane (μ, λ) is decomposed into "existence" ("good") and "non-existence" ("bad") subsets. This decomposition can be made provided that the Fučík spectrum is known.

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NON-LINEAR FINITE ELEMENT MODELLING OF FLAX SHORT FIBER REINFORCED AND FLAX FIBRE FABRIC REINFORCED POLYMER COMPOSITES

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The ever-increasingly demand of natural fiber reinforced polymer composites in various engineering applications calls for accurate predictions of their mechanical behaviours. In this study, numerical methods to generate and simulate mechanical properties of flax short fiber reinforced and flax fabric reinforced-polymer composites are proposed. Methods to simulate a quasi-static behaviour and dynamic behaviour is presented. Quasi-static tension tests are performed using non-linear finite element modelling. Natural fiber reinforced composites have a good energy absorbing properties. In this study a crash test of empty and PU foam filled flax-epoxy tubes are presented. The microstructures of short flax fibres with different fibre length-to-diameter ratios are generated by algorithm taking fiber defects (e.g. kink band) and fiber bundles into account. A brittle material law for fiber defects and interfacial zones of fiber bundles is proposed. Flax short fibre/polypropylene and flax fabric/epoxy composites are modelled by a non-linear plasticity model considering an isotropic hardening law and non-local continuum damage mechanics. This study shows that the simulation can capture the main damage mechanisms of the composites such as fiber breakage initiated at the fiber defects, damage of polymer matrix and the fibre debonding at fibre/matrix interface accurately. In addition, the simulation results exhibit good agreements with the experimental results in the aspects of elastic properties and non-linear tensile stress-strain behaviour of the short fibre and fibre fabric reinforced polymer composites up to ultimate strains (see Figure 1 [1]).

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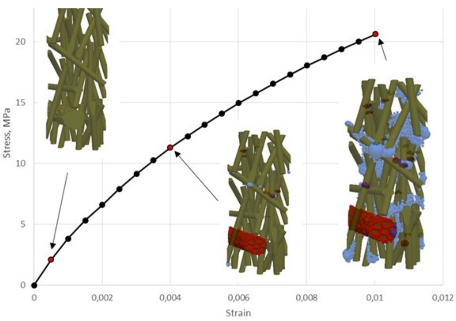

Figure 1. Stress-strain curve and damage-plastic deformation field of short flax fiber reinforced polymer.

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ON THE AREAS UNDER THE OSCILLATORY CURVES

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Behavior of antiderivatives of solutions for ordinary differential equations plays an important role in the theory of nonlocal boundary value problems [3]. Results on the estimation of the number of solutions to boundary value problems with integral conditions often are related with the oscillatory properties of solutions and its antiderivatives [2].

We prove that the third order Emden-Fowler type equation

$$x''' = -|x|^p \operatorname{sign} x \tag{1}$$

has oscillatory solutions with specific behavior of antiderivative.

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DYNAMIC EQUATIONS ON TIME SCALES

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The dynamic equations on time scales unify continuous and discrete analysis. A time scale $\mathbb{T}$ is an arbitrary closed subset of the real numbers. This talk will be introduction to the dynamic equations on time scales. The basic definitions - a time scales, forward jump operator, backward jump operator, scattered, dense, isolated points, differentiation etc. and examples will be introduced. The time scales calculus have a great potential of application.

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EQUITATION FROM DYNAMICAL SYSTEMS PERSPECTIVE

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This talk is a contribution towards mathematical description of dressage equitation. This process involves the interaction of a human and an equine. Certain aspects of this process are way out of reach for the current techniques of mathematical modelling, but certain biomechanical constraints in the gaits and transitions are not.

The gaits of quadrupeds can be classified using various systems, some of them use measurable quantitative indicators, e.g., the Hildebrand grid [1] uses the pair *body support time fraction on (left) hind leg* and *advanced lateral placement of the (left) foreleg*. This reduces more complex models of equine kinematics to just a couple of numerical parameters. Hildebrand grid has been extended to asymmetrical gaits in [2]. Some studies have been devoted to the origin of the gaits, and minimal size of the central pattern generator has been found by studying symmetries of periodic solutions to equivariant dynamical systems yielding the observed gaits and transitions, cf. [3]. It is also noted that the gaits and transitions are chosen by the animal according to its current needs.

The interaction between horse and rider is hard to model. It involves not just biomechanical interaction, but also learning in both partners, involving information flow disturbed by dynamics of the horse-rider system. An overview of the qualitative features of the problem can be found in [4]. Several studies have focused on extracting measurable data from observations of horse-rider dyads, see e.g. [5].

In the spirit of the program outlined in [2], we propose a model of a dressage test as a dynamical system with states on Hildebrand grid or related phase space.

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ON SOME DISCRETE EPIDEMIC MODELS

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In [1] are formulated two research problems (Research Project 6.7.1. and 6.7.2.) about the difference equation with an exponent

$$x_{n+1} = \left(1 - \sum_{j=0}^{k-1} x_{n-j}\right)(1 - e^{-Ax_n}), \quad n = 0, 1, ..., \tag{1}$$

which is a discrete epidemic model.

In [2] authors proposed Open Problem 6.10.14 about the difference equation

$$x_{n+1} = (1 - x_n - x_{n-1})(1 - e^{-Ax_n}), \quad n = 0, 1, ... \tag{2}$$

In [4] authors study the oscillation, global asymptotic stability, and other properties of the positive solutions of the difference equation (1). In [3] authors investigate the global stability of the negative solutions of (1).

We investigate a difference equation

$$x_{n+1} = (1 - x_n - x_{n-1})(1 - e^{-Ax_n - Bx_{n-1}}), \quad n = 0, 1, ... \tag{3}$$

where $A, B > 0$ and the initial values x_{-1}, x_0 are arbitrary real positive numbers such that $x_{-1} + x_0 < 1$.

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REDUCTION OF THE RESONANT BOUNDARY VALUE PROBLEM TO A QUASILINEAR PROBLEM

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We study a resonant boundary value problem

$$x'' + k^2 x = f(t, x), \quad x(0) = 0,\ x(1) = 0. \tag{1}$$

The quasilinearization approach of [2] is used. The problem is made regular (non-resonant) by adding the same linear term to the left and right hand sides. A non-resonant linear part appears on the left side. The problem looks like

$$x'' + k^2 x + \varepsilon^2 x = f(t, x) + \varepsilon^2 x =: F(t, x), \quad x(0) = 0, \quad x(1) = 0. \tag{2}$$

The right side F in (2) is then replaced by a bounded function $F_1(t, x) = F(t, \delta(-N, x, N))$, where $\delta(-N, x, N)$ is a truncation (cut-off) function. The given problem becomes non-resonant quasilinear one of the form

$$x'' + k^2 x + \varepsilon^2 x = F_1(t, x), \quad x(0) = 0, \quad x(1) = 0. \tag{3}$$

This problem may be shown to have a solution if the estimate

$$\Gamma \cdot M \leq N \tag{4}$$

holds, where Γ and M are respectively bounds (estimate constants) for the Green's function $|G|$ that corresponds to a modified non-resonant linear part (and given boundary conditions) and for the right hand side $|F_1(t, x)|$ in (3). Then $|x(t)| \leq N$ for any $t \in [0, 1]$ and $x(t)$ is also a solution of (1). This scheme was tested on equations of the Emden-Fowler type in [1]. The appropriate choice of ε is relatively small on an interval $[-N, N]$. Then a constant M in (4) is relatively small also and the inequality (4) is likely to be satisfied.

Our intention is to extend this technique to more general cases. Two ways of relaxing the inequality (4) are proposed and discussed. First, to modify a resonant problem to a regular one we use the Taylor expansion for $f(t, x)$ with respect to x. The second way of relaxing the inequality (4) is based on an appropriate choice of "good" approximation to expected solution. We provide the existence results illustrating both ways.

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M-VALUED BORNOLOGIES ON L-VALUED SETS

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We develop an approach to the concept of bornology [3] in the framework of many-valued mathematical structures. It is based on the introduced here concept of an M-valued bornology on an L-valued set (X, E), where L is a GL-monoid [2], M is a complete completely distributive lattice and $E : X \times X \to L$ is an L-valued equality [2] on the set X. We develop the basics of the theory of LM-valued bornological spaces, that is triples $(X, E, \mathcal{B})$ where $\mathcal{B}$ is an M-valued bornology on the L-valued set (X, E). We initiate the study of the category of LM-valued bornological spaces and appropriately defined bounded "mappings" of such spaces. A scheme for constructing M-valued bornologies on L-valued sets with prescribed properties is developed. In particular, it allows to extend an ordinary bornology on a set to an M-valued bornology on it. This scheme is based on the use of LM-valued equalities generalizing "ordinary" L-valued equalities.

In the special case when E is crisp, that is $E(x, x') = 1$ iff $x = x$ and $E(x, x') = 0$ otherwise our theory of M-valued bornologies on L-valued sets contains, as special case, the approaches presented in [1] and [4].

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TREE EDIT DISTANCE PROBLEM, PARALLELIZABLE APPROACH

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The problem of transforming or editing trees is a means of tree comparison, with applications in combinatorial pattern matching [1].

In general sense trees can be transformed by application of elementary edit operations which might have a weight function associated. So edit distance between trees is the weight of least-weight sequence of elementary operations that transforms one tree into another. Only labeled rooted ordered trees are taken into consideration. Elementary edit operations include: removal of vertex, insertion of vertex and renaming of vertex [2].

The parallelizable divide-and-conquer approach can be applied to tree edit distance problem. This is possible because problem exhibits independence principle, meaning a problem instance can be divided into smaller problem instances, which in turn can be solved independently [3] .

Report deals with formulation and construction of divide-and-conquer subproblem tree and answer retrieval.

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GENERALIZED ADDITIVE MODELS FOR LATVIAN WATER UTILITY DATA

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Generalized additive models (GAM) have obtained large popularity among statisticians mainly due the nonparametric nature. These models were introduced by Hastie and Tibshirani [1] to blend properties of generalized linear models with additive models. The main difference between the linear (or nonlinear regression methods) is that for GAM the data determine the best functional dependence form using some appropriate smoothing methods. In this case the impact of all the predictors is analyzed by some smooth functions in a linear fashion. Smooth terms are represented by regression splines or smooth cubic splines. The fitting of the generalized additive model using the R package `mgcv` ensures the opportunity to evaluate the statistical significance and to derive confidence bands of smooth functions of predictors (for reference see [2]). We discuss several applications to ecological data and Latvian water utility data.

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SMOOTHED JACKKNIFE EMPIRICAL LIKELIHOOD INFERENCE FOR ROC CURVES

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The receiver operating characteristic (ROC) curve was first introduced by electrical and radar engineers during the World War II for detecting enemy objects in battlefields. Since then it is a popular method for the accuracy of diagnostic tests, it is broadly used in medical studies, machine learning, decision making, etc.

Let F_D be the distribution function of the diseased subjects, and by $F_{\bar{D}}$ denote the distribution function of non-diseased subjects. Then the true positive rate at a fixed threshold c will be $TPR(c) = 1 - F_D(c)$ and false positive rate $FPR(c) = 1 - F_{\bar{D}}(c)$. A ROC curve is defined as a set of points $\{FPR(c), TPR(c)\}$.

The quantitative assessment of the accuracy of a diagnostic test is the area under the ROC curve (AUC). The partial area under the ROC curve (partial AUC) summarizes the accuracy of a diagnostic test over a relevant region of the ROC curve and represents a useful tool for the evaluation and comparison of tests.

There are many parametric and nonparametric methods for the estimation of ROC curves and AUC. Recently the empirical likelihood method has been extensively used for the estimation and the construction of confidence intervals for ROC curves. Empirical likelihood (EL) was first introduced by Owen [4]. Later, other researchers expanded empirical likelihood methodology to the ROC curves [2] and for the partial area under the ROC curve. The jackknife empirical likelihood (JEL) method has received more attention because it improves the computational efficiency successfully by reducing nuisance parameters. Gong et al. [3] applied the smoothed jackknife empirical likelihood method for ROC curves and Zhao [5] developed the smoothed jackknife empirical likelihood (SJEL) inference for the difference of ROC curves. JEL methods demonstrated some advantages in comparison with the existing EL methods based on computational issues. Whereas Adimari [1] presented the JEL method for partial AUC and the difference of two partial AUC.

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ON QUASI-LINEAR PROBLEM WITH PERIODIC BOUNDARY CONDITIONS

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We consider the second order differential equation of the form

$$X'' = AX + H(X)\,, \tag{1}$$

where $X \in C^2\big([0, +\infty; \mathbb{R}^n\big)$, $A \in \mathbb{R}^{n\times n}$, and function $H : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is continuous and bounded with a property $H(\mathcal{O}) = \mathcal{O}$ ($\mathcal{O} = (0, 0, \ldots, 0) \in \mathbb{R}^n$), subject to the periodic boundary conditions

$$X(0) = X(T), \qquad X'(0) = X'(T) \qquad (T > 0). \tag{2}$$

The theory of vector fields is used to obtain conditions for solvability of the problem (1), (2).

We discuss the conditions under which the problem (1), (2) has nonconstant solutions and investigate estimations of appropriate $T > 0$.

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ON MINIMAL RIGHT TRIANGLE CIRCUMSCRIBING CIRCULAR AND PARABOLIC SEGMENT

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Let S be a convex plane shape with finite positive area $A(S)$. Prove (or disprove) that S can be included in such a right-angled triangle T that

$$A(T) \leq 2A(S).$$

This problem generally speaking is unsolved but some special cases were solved by several authors, see for example [1] and [2]. Problem is also solved if there are no restrictions for enclosing triangle, see [3].

We consider this problem when shape S is circular segment or parabolic segment (with axial symmetry). We show that there are such constants c_1 and c_2 from the interval $(1, 2)$ that any circular segment C and any symmetric parabolic segment P can be included in such right-angled triangles T_1 and T_2 that $A(T_1) \leq c_1 A(C)$ and $A(T_2) \leq c_2 A(P)$. We also consider convex plane shapes consisting of two parabolic segments.

Next, we consider a generalization of this problem, namely, n-dimensional shape S will be included in a simplex with an "orthogonal corner". Author has hypothesis that shape S with n-dimensional volume $V(S)$ can be included in a simplex F with an "orthogonal corner" such that $V(F) \leq \dfrac{n^n}{n!} \cdot V(S)$.

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ABOUT NEURON MODEL WITH PERIODIC WITH PERIOD TWO INTERNAL DECAY RATE

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In [1] a difference equation $x_{n+1} = \beta x_n - g(x_n)$, $\quad n = 0, 1, 2, ...$, was analyzed as a single neuron model, where $\beta > 0$ is an internal decay rate and a signal function g is the following piecewise constant function with McCulloch-Pitts nonlinearity:

$$g(x) = \begin{cases} 1, & x \geq 0, \\ -1, & x < 0. \end{cases} \tag{1}$$

Now we will study the following non-autonomous piecewise linear difference equation:

$$x_{n+1} = \beta_n x_n - g(x_n), \quad n = 0, 1, 2, ...,$$

where $(\beta_n)_{n=0}^{\infty}$ is a period three sequence

$$\beta_n = \begin{cases} \beta_0, & \text{if } n = 2k, \\ \beta_1, & \text{if } n = 2k+1, \end{cases} \quad k = 0, 1, 2, ...$$

and g is in form (1).

The goal of this paper is to investigate the boundedness nature and the periodic character of solutions. Furthermore, we will determine the relationships of the periodic cycles relative to the periods of the parameters and relative to the relationship between the parameters as well. Moreover, we will investigate which particular periodic cycles can be only periodic and which particular periodic solutions can be eventually periodic. In addition, we will show the bifurcation diagrams when solutions transition from periodicity of various periods to unbounded solutions.

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