Properties of polynomials and problems in differential equations

Felix Sadyrbaev
Daugavpils University
Daugavpils, Latvia

The famous problems [1, Ch. 3] deal with systems of differential equations of the form

$$\begin{aligned} x' &= P(x,y), \\ y' &= Q(x,y), \end{aligned} \tag{1}$$

where P and Q are polynomials. The second part of 16th Hilbert problem asks for a number of limit cycles in systems of the form (1). The limit cycle is an isolated periodic solution $(x(t), y(t))$ of the system (1). The expected solution of the Hilbert's problem depends on properties of polynomials P and Q.

In our talk we consequently consider the first part of the Hilbert's 16th problem concerning the maximal possible numbers of connected components that an algebraic curve can have, in terms of the degree of the curve.

The second part deals with the maximal possible number of limit cycles in the system of the form (1), in terms of the degrees of polynomials P and Q. We give brief account on the current state of the problem.

Next we treat the Smale's problem which concerns the specific case of the system (1), namely, the Liénard type equation

$$x'' + f(x)x' + x = 0. \tag{2}$$

Finally, we present recent results on the number and location of limit cycles in the generalized Liénard type equation

$$x'' + f(x, x')x' + g(x) = 0, \tag{3}$$

where f and g are polynomials.

References

[1] L. Perko. *Differential Equations and Dynamical Systems, 3rd ed.,* Texts in Applied Mathematics 7, Springer, 2001.