On endomorphism monoids of Clifford semigroups

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A. H. Clifford [1] characterized the semigroups that *admit relative inverses*. A semigroup S admits relative inverses if it satisfies the following condition: to each $a \in S$ there exist $e, b \in S$ such that $ea = ae = a$ and $ab = ba = e$. Later these semigroups were called *Clifford semigroups*. The construction of Clifford semigroups is the following.

Let Λ be a semilattice, i. e. a commutative idempotent semigroup. The partial order on Λ is determined by the multiplication as follows: if α, $\beta \in \Lambda$, then $\beta \leq \alpha$ if and only if $\alpha\beta = \beta$. A Clifford semigroup is a disjoint union of groups $S = \bigsqcup_{\alpha \in \Lambda} G_\alpha$ indexed by a semilattice Λ, together with a group homomorphism $\varphi_{\alpha,\beta} : G_\alpha \longrightarrow G_\beta, \quad \alpha \geq \beta$ whenever $\alpha \geq \beta$ in Λ, such that 1) for each $\alpha \in \Lambda$, the homomorphism $\varphi_{\alpha,\alpha}$ is the identity, 2) if $\alpha \geq \beta \geq \gamma$ then $\varphi_{\alpha,\gamma} = \varphi_{\alpha,\beta}\varphi_{\beta,\gamma}$. The semigroup operation on S is defined by $ab = (a\varphi_{\alpha,\alpha\beta}) \cdot (b\varphi_{\beta,\alpha\beta})$ where $a \in G_\alpha$ and $b \in G_\beta$.

M. Samman and J. D. P. Meldrum [3] gave a characterization of endomorphisms of Clifford semigroups. Seminear-rings of endomorphisms of Clifford semigroups were studied by N. D. Gilbert and M. Samman [2].

In our talk, we discuss the problem of the unique recoverability of Clifford semigroups by their endomorphism monoids.

References

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[2] N. D. Gilbert and M. Samman, Clifford semigroups and seminear-rings of endomorphisms, *Intern. Electronic J. of Algebra* 7, 2010, 110–119.

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