# Categorical invariants of finite minimal majority algebras

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The algebras $\mathbf{A}$ and $\mathbf{B}$ are called *categorically equivalent* if there is an equivalence between categories $\operatorname{Var}\mathbf{A}$ and $\operatorname{Var}\mathbf{B}$ under which the algebras $\mathbf{A}$ and $\mathbf{B}$ correspond to each other. We denote by $\mathbf{A}^+$ the algebra obtained from an algebra $\mathbf{A}$ by adding to its fundamental operations all nullary operations on $A$. We shall call algebras $\mathbf{A}$ and $\mathbf{B}$ *p-categorically equivalent* if $\mathbf{A}^+$ and $\mathbf{B}^+$ are categorically equivalent.

A property $P$ is called a *categorical* (*p-categorical*) *invariant* for a class of algebras $\mathcal{K}$, if for any two categorically (p-categorically) equivalent algebras $\mathbf{A}, \mathbf{B} \in \mathcal{K}$, algebra $\mathbf{A}$ has the property $P$ iff so does $\mathbf{B}$.

We call an algebra *minimal* if it has no proper subalgebras. Obviously all algebras $\mathbf{A}^+$ are minimal. A *majority algebra* is one that admits a majority term. It follows from [1] that categorical invariants of finite majority algebras $\mathbf{A}$ are determined by the structure $\mathbf{S}_2(\mathbf{A})$ of subalgebras of $\mathbf{A}^2$. For the case with $\mathbf{A}$ minimal and $\operatorname{Var}\mathbf{A}$ arithmetical, the structures $\mathbf{S}_2(\mathbf{A})$ where described in [2]. Later this result was generalized to arbitrary finite minimal algebras with majority term (not published yet). It turned out that in this case the structures $\mathbf{S}_2(\mathbf{A})$ ($\mathbf{S}_2(\mathbf{A}^+)$) are precisely finite, ordered, involutive monoids with zero, satisfying some natural distributivity and completeness conditions (finite, involutive, lattice ordered semirings). Using these results we can derive a number of categorical invariants of finite, minimal majority algebras and p-categorical invariants of finite majority algebras.

## References

[1] C. Bergman, Categorical equivalence of algebras with majority term *Algebra Universalis* 40, 1998, 149–175.

[2] K. Kaarli and L. Márki, A characterization of the inverse monoid of bi-congruences of certain algebras, *International J. Algebra and Computation* 6, 2009, 791–808.