AGGREGATION OPERATORS AND T-NORM BASED OPERATIONS WITH L-FUZZY REAL NUMBERS

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Our work deals with a fuzzy analogue of a real number. In the literature on fuzzy mathematics one can find several different schemes for defining fuzzy numbers. We consider the notion originating from B. Hutton paper [1] and later developed by several authors.

Let \( L = (L, \wedge, \vee) \) be a completely distributive lattice with lower and upper bounds \( 0_L, 1_L \in L \). An \( L \)-fuzzy real number is a function \( z : \mathbb{R} \to L \) such that

(i) \( z \) is non-increasing;
(ii) \( \bigwedge_{x} z(x) = 0_L, \bigvee_{x} z(x) = 1_L \);
(iii) \( z \) is left semi-continuous, i.e. \( \bigwedge_{t < x} z(t) = z(x) \).

The set of all fuzzy real numbers is called the fuzzy real line and is denoted by \( \mathbb{R}(L) \). The operations of \( L \)-fuzzy addition and \( L \)-fuzzy multiplication as they are defined in [2] are jointly continuous extensions of addition and multiplication from the real line \( \mathbb{R} \) to the \( L \)-fuzzy real line \( \mathbb{R}(L) \).

The aim of this talk is to present alternative definitions for arithmetic operations with \( L \)-fuzzy numbers which are based on a triangular norm (recall that a triangular norm, or a \( t \)-norm for short, is an associative, commutative binary operation on a lattice \( L \) which is non-decreasing in each argument and has the neutral element \( 1_L \) [3]). For this aim we use the \( t \)-norm extension \( \hat{A} \) of an aggregation operator \( A \) which is defined by the following formula [4]:

\[
\hat{A}(z_1, \ldots, z_n)(x) = \bigvee_{x = A(x_1, \ldots, x_n)} T(z_1(x_1), \ldots, z_n(x_n)),
\]

where \( z_1, \ldots, z_n \in \mathbb{R}(L), x_1, \ldots, x_n \in \mathbb{R} \) and \( T \) is a \( t \)-norm. Basic algebraic properties of these arithmetic operations are discussed. Examples illustrating the role of a \( t \)-norm in the definition of operations are given. In particular we consider the cases of minimum, product and Lukasiewicz \( t \)-norms.

REFERENCES