ON A CATEGORY OF L-TOPOLOGICAL SPACES ON GLOBAL L-VALUED SETS

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Let $L = (L, \leq, \land, \lor, \ast)$ be a GL-monoid [1]. A global $L$-valued equality on a set $X$ is a mapping $E : X \times X \to L$ such that:

1) $E(x, x) = 1 \ \forall x, y \in X$;
2) $E(x, y) = E(y, x) \ \forall x, y \in X$;
3) $E(x, y) \ast E(y, z) \leq E(x, z) \ \forall x, y, z \in X$.

An $L$-valued set is a pair $(X, E)$ where $X$ is a set and $E$ is a global $L$-valued equality on it.

An $L$-subset $A$ of an $L$-valued set $(X, E)$ is called extensional if

$$\bigvee_{x \in X} A(x) \ast E(x, x') \leq A(x') \ \forall x' \in X.$$

Let $L(X)$ denote the family of all extensional $L$-subsets of $X$. By an $L$-fuzzy topology on a global $L$-valued set $(X, E)$ we call a mapping $T : L(X) \to L$ s. t.

1) $T(1_X) = T(0_X) = 1$;
2) $T(U \land V) \geq T(U) \land T(V) \ \forall U, V \in L(X)$;
3) $T\left(\bigvee_{i \in I} (U_i)\right) \geq \bigwedge_{i \in I} T(U_i) \ \forall \{U_i \mid i \in I\} \subset L(X)$.

The triple $(X, E, T)$ is called an $L$-fuzzy $L$-valued topological space.

Note that in case $E$ is crisp the above definition reduces to the definition of an $L$-fuzzy topological space in the sense of [2].

Let $L\text{-FTOP}(L)$ denote the category whose objects are $L$-fuzzy $L$-valued topological spaces and whose morphisms are extensional continuous mappings between them. (The continuity of $f : (X, E_X, T_X) \to (Y, E_Y, T_Y)$ means that $T_X(f^{-1}(V)) \geq T_Y(V)$ for each $V \in (L)(X)$.)

Our aim here is to discuss some properties of the category $L\text{-FTOP}(L)$ and its objects. In particular, it will be shown that $L\text{-FTOP}(L)$ is topological over the category $\text{SET}(L)$ of global $L$-valued sets. Besides some relations between this category and some other categories will be studied.

REFERENCES
